Bound states for the Schrödinger equation with mixed-type nonlinearities

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We investigate the existence and multiplicity of solutions for the Schrödinger equation of the form

$$-\Delta u + V(x)u = g(x,u), \quad x \in \mathbb{R}^N, \ u \in H^1(\mathbb{R}^N).$$

In nonlinear optics, this equation describes the propagation of a electromagnetic wave in a periodic waveguide, e.g. photonic crystals. The external potential V takes into account the linear properties of the material and as usual we assume that V is $\mathbb{Z}^N$-periodic and 0 lies in the spectral gap of the Schrödinger operator $-\Delta + V(x)$. The nonlinear function g is responsible for the polarization of the medium. For instance, in Kerr-like media one has $g(x,u) = |u|^2 u$ and in the saturation effect g is asymptotically linear and is of the form $g(x,u) = \frac{|u|^2}{1+|u|^2}u$. Recently it has been shown that materials with large range of prescribed properties can be created and our aim is to model a wide range of nonlinear phenomena that allow to consider a composite of materials with different nonlinear polarization. In our case g may be linear for some $\mathbb{R}^N \setminus K$ (for sufficiently large $|u|$) and nonlinear outside of it, where K is a given $\mathbb{Z}^N$-periodic set. In particular we may combine the Kerr-like nonlinearity with a saturation effect. Under our conditions the energy functional has the linking geometry and Cerami sequences are bounded. This allows to use a variant of linking theorem to obtain the existence of solutions. However, the multiplicity of solutions seem to be difficult to obtain by standard methods. Hence we reduce the problem to an appropriate subspace of $H^1(\mathbb{R}^N)$, where the quadratic form is positive-definite and use a Cerami-type condition, and a variant of Benci's pseudoindex to show the multiplicity of solutions. In fact we refine a recent critical point theory from [2] for strongly indefinite functionals which do not have to be globally super-quadratic.

References

1 B. Bieganowski, J. Mederski: *Bound states for the Schrödinger equation with mixed-type nonlinearities*, arXiv: 1905.04542

2 J. Mederski, J. Schino, A. Szulkin *Multiple solutions to a nonlinear curl-curl problem in* $\mathbb{R}^N$, arXiv:1901.05776.