Homological properties of Schreier extensions of monoids

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In order to extend to monoids the classical equivalence between group actions and split extensions, the notion of Schreier split extension of monoids was introduced. Such split extensions are those which correspond to monoid actions, where an action of a monoid B on a monoid X is a monoid homomorphism $B \to \operatorname{End}(X)$.

Unlike general monoid extensions, Schreier (split) extensions have strong homological properties. In particular, the classical homological lemmas that are valid for group extensions, such as the Short Five Lemma and the Nine Lemma, hold for Schreier extensions. A particular role is played by the so-called *special* Schreier extensions, namely those whose kernel is a group. Indeed, a special Schreier extension $f: A \to B$ of monoids, with abelian kernel X, determines a monoid action of B on X. This fact allows to make a partition of the set SchExt(B, X) of special Schreier extensions of the monoid B by the abelian group X into subsets of the form $SchExt(B, X, \varphi)$ of special Schreier extensions inducing the same action $\varphi: B \to End(X)$, in analogy with what happens for group extensions.

We first give a description of Baer sums in terms of factor sets: we show that special Schreier extensions with abelian kernel correspond to equivalence classes of factor sets, as it happens for groups. Pointwise multiplication of factor sets induces then an abelian group structure on any set $SchExt(B, X, \varphi)$.

Secondly, we introduce a push forward construction for special Schreier extensions with abelian kernel. This construction allows us to give an alternative, functorial description of the Baer sum, opening the way to an interpretation of the cohomology of monoids with coefficients in modules (which is a generalization of the classical Eilenberg-Mac Lane cohomology of groups) in terms of extensions.

Another advantage of this functorial approach is that it can be extended to general Schreier extensions, without the assumption that the kernel is a group. We describe a classification of all such extensions in cohomological terms.