Koszul Property in Galois Cohomology 2

- The Return -

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13.30 - 20/12/2018 Università di Milano-Bicocca Building U5 - Room 3014

Koszulity is back in Galois cohomology. And it is stronger than ever!

Absolute Galois groups of fields are a main object of interest in algebraic number theory and related subjects, but we are very far from a satisfactory understanding of their structure. Yet recently a great breakthrough has been obtained with the proof of the Bloch-Kato conjecture. This gave mathematicians the first substantial insight on the rather mysterious Galois cohomology of an absolute Galois group (and of its pro-p quotients), an important invariant of a field. Its most significant consequence is that, in case a field contains a primitive p-th root of unity, the Galois cohomology of its absolute Galois group is a quadratic algebra. There is a class of quadratic algebras with an uncommonly good homological behaviour and endowed with a useful duality functor: the class of Koszul algebras. L. Positselski conjectured that in the above situation the Galois cohomology of absolute Galois groups is always Koszul, and proved this for various classes of fields, e.g. for algebraic number fields.

In a previous talk we discussed the meaning of Koszulity in the framework of Galois theory and we showed that the Galois cohomology of a pro-p group of elementary type is Koszul. After recalling the background of this old result, we introduce various stronger versions of the Koszul property and we present new theorems showing that nearly all of these stronger forms of Koszulity hold for the Galois cohomology of a pro-p group of elementary type. Each of these properties add deep insights on the homological behaviour of modules over those algebras. This is a joint work with Ján Mináč, Marina Palaisti and Nguyễn Duy Tân.

Keywords:

Algebraic number theory • Galois groups • Quadratic and Koszul algebras • Syzygies, resolutions, complexes • Witt groups and rings

"Obvious" is the most dangerous word in mathematics. - Eric Temple Bell