Insalate di Matematica presents

Two scale-convergence method

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Many applications, such as porous media or composite materials, involve heterogeneous media described by partial differential equations with coefficients that randomly vary on a small scale. On macroscopic scales (large compared to the dimension of the heterogeneities) such media often show an effective behavior. Typically that behavior is simpler, since the complicated, random small scale structure of the media averages out on large scales, and in many cases the effective behavior can be described by a deterministic, macroscopic model with constant coefficients. This process of averaging is called homogenization. Kozlov [Koz1979], Papanicolaou and Varadhan [Pap1979] studied (steady) heat conduction in a randomly inhomogeneous conducting medium and obtained a qualitative homogenization result for stationary, ergodic conductivities. They proved that in the homogenization limit an effective conductivity emerges and it is described by the homogenization formula

$$e\mathbf{a}_{hom}e := \int_{\Omega} (e + \nabla\phi)\mathbf{a}(0)(e + \nabla\phi)d\mu \quad \text{for all } e \in \mathbb{R}^d.$$

The homogenization formula involves a function ϕ which is called corrector along $e \in \mathbb{R}^d$. In order to characterize ϕ in a variational setting, we introduce the notion of two scale-convergence. The aim of this talk is to describe the asymptotic behavior as $\varepsilon \to 0$ of the following problem:

$$\begin{aligned} -div(A(\frac{x}{\varepsilon})\nabla u_\varepsilon) &= f \quad \text{in } D \\ u_\varepsilon &= 0 \quad \text{on } \partial D, \end{aligned}$$

where $D \subset \mathbb{R}^d$ is bounded and $f \in H^{-1}(D)$.

"Obvious" is the most dangerous word in mathematics.
- Eric Temple Bell