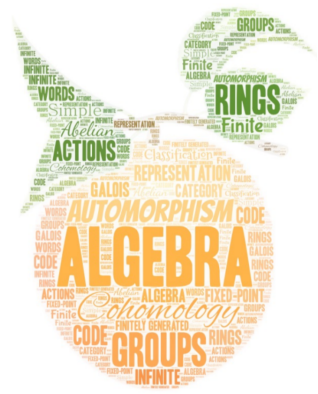# AI@Bicocca seminar

You are all welcome to the next bite of the series
"**AI@Bicocca**"
which is meant to give you a small taste of the Algebra at Bicocca and beyond

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Filtrations on Graph Products: Applications in Galois Theory and Geometry

Abstract: Graph products, along with their filtrations and cohomology rings, are central to geometric group theory and have recently found significant applications in Galois theory.

In this talk, I will first present recent results by Maire, Minac, Tân, and myself on Pythagorean fields. In the same spirit as the work of Snopce-Zalesskii and Blumer-Quadelli-Weigel, we introduce, using Right Angled Artin groups, a new necessary condition for absolute Galois groups. The main arguments come from filtration techniques and Minac's PhD thesis. As an application, we exhibit the first example of a pro-2 group that is not the maximal pro-2 quotient of an absolute Galois group but satisfies both the Bloch-Kato and Minac-Tân conjectures (on filtrations and cohomology rings).

In the second part, I will present my recent results on filtrations of graph products, continuing the work of Panov-Rahmatullaev, Veryovkin, Leoni-Weigel and Marmo-Riley-Weigel. I begin by providing a positive solution to a question posed by Minac, Rogelstad, and Tân regarding the relationship between the Zassenhaus and lower central series of pro-p groups under a torsion-freeness condition. This condition is generally difficult to verify and is only known to hold in a few cases. Independently, a relation between the two previous filtrations was also investigated by Marmo-Riley-Weigel. I show that graph products of (pro-p) groups behave well with respect to this torsion-freeness condition. Additionally, I compute the graded Lie algebras associated with these filtrations under natural assumptions. Specifically, I apply all of these results to graph products of fundamental groups of surfaces, providing new examples that satisfy the torsion-freeness condition from the question of Minac, Rogelstad, and Tân.