Book of abstract

Young Reasearcher Meeting on PDEs

September, 25-27 2024

Università degli Studi di Milano-Bicocca

Building Atlas (Ex-U1) Room U1-07 Aula Marchetti

Periodo: 25-26-27 Settembre 2024.

Sede ospitante: Università di Milano-Bicocca.

Comitato scientifico e organizzatore Federico Bernini (Università di Milano), Andrea Bisterzo (Università di Milano-Bicocca), Nicolò De Ponti (Università di Milano-Bicocca), Paolo Malanchini (Università di Milano-Bicocca), Claudia Nocito (Università di Milano-Bicocca), Roberto Ognibene (Università di Pisa), Giovanni Siclari (Università di Milano-Bicocca).

Finanziatori: GNAMPA-INDAM, Università di Milano-Bicocca, Università di Milano Statale.

Descrizione dell'evento

Il presente convegno ha lo scopo di riunire giovani ricercatori (*dottorandi, assegnisti, post-doc*), provenienti sia da Università italiane che straniere, che si interessano allo studio, mediante differenti teorie, di equazioni differenziali alle derivate parziali (PDE) di natura ellittica e/o parabolica.

Il convegno è organizzato da giovani per giovani: l'idea è di offrire un ambiente stimolante che aiuti nella crescita formativa personale, promuovendo la collaborazione e interazione tra le nuove generazioni di matematici. Inoltre, puntiamo a fornire un ambiente "protetto" in cui poter accrescere e migliorare le competenze espositive.

Il comitato scientifico e organizzatore, composto unicamente da dottorandi e assegnisti, mira a garantire un'ampia varietà di argomenti trattati, quali: *esistenza di soluzioni di PDE ellittiche e quasi-lineari, proprietà qualitative di PDE ellittiche, principi di monotonia, aspetti geometrici, applicazioni alla Fisica Matematica, problemi di regolarità e free-boundary, aspetti spettrali, mean field-games, equazioni di evoluzione,...*

Inoltre, la commissione avrà la premura di assicurare la parità di genere tra gli speaker invitati.

UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI MATEMATICA "FEDERIGO ENRIQUES"

DIPARTIMENTO DI MATEMATICA E APPLICAZIONI

GRUPPO NAZIONALE PER L'ANALISI MATEMATICA, LA PROBABILITÀ E LE LORO APPLICAZIONI

GRUPPO DI ANALISI NONLINEARE 2021-ATE-0253

SEPTEMBER 25-27, 2024

YOUNG RESEARCHERS MEETING ON PDES

Università degli studi di Milano-Bicocca
Milano - Italy

SPEAKERS:

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Università Politecnica delle Marche

Luigi Appolloni
University of Leeds

Laura Baldelli
Universidad de Granada

Chiara Bernardini
Università di Trento

Annamaria Massimini
TU Wien, ESILV

Giulia Bevilacqua
Università di Pisa

Gabriele Cora
Università di Torino

Alessandra De Luca
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Lorenzo Liverani
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Ludovico Marini
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Roberto Ognibene (Università di Pisa)
Giovanni Siclari (Politecnico di Milano)

Contents

Regularity results for kinetic equations: local vs. nonlocal behavior

Francesca Anceschi - Università Politecnica delle Marche

Abstract

In this talk, we present some recent results on the weak regularity theory for kinetic equations. We will focus on Harnack inequalities and their different formulation in the context of local, or nonlocal, diffusion.

Some existence results for a non-local critical p-Kirchhoff equation

Luigi Appolloni - University of Leeds

Abstract

In 1897, Kirchhoff introduced a model aimed at generalizing the well-known d'Alembert equation by also incorporating lateral displacement. Over the years, Kirchhoff's equation has been extended in various directions, as it proved capable of modeling a wide range of phenomena. In this talk, we will present some recent existence results for a non-local, nonlinear version of the Kirchhoff equation, driven by the fractional p-Laplacian and featuring a critical term in the sense of the Sobolev exponent on the right-hand side. These results can be viewed as an extension of those published by Brezis and Nirenberg in their seminal 1983 paper.

A quasilinear problem with a serious lack of compactness

Laura Baldelli - Universidad de Granada

Abstract

In this talk, we will disclose the main result contained in a recent paper written jointly with Umberto Guarnotta (Università Politecnica delle Marche, Italy), where we proved the existence of a weak solution to a p-Laplacian problem set in the whole Euclidean space exhibiting a critical term perturbed by a singular and convective reaction.

Our ambition comes from the fact that the problem under consideration mixes some variational, where a dangerous lack of compactness of Sobolev's embedding occurs due to the presence of the Sobolev critical exponent and the setting in the entire Euclidean space, and non-variational problems since convection terms destroy the variational structure. Moreover, the singularity prevents us from using direct classical variational techniques. The multi-faced aspect of the problem suggests that it may be of interest in describing, forecasting, and controlling the evolution of a variety of phenomena in physics, chemistry, finance, biology, and medicine. The approach used combines variational methods, truncation techniques, and concentration compactness arguments, together with set-valued analysis and fixed point theory. Moreover, we study local $C^{1,\alpha}$ regularity of solutions, as well as their pointwise decay at infinity, by De Giorgi's technique, a priori gradient estimates and nonlinear regularity theory. As far as we know, the result is new even in the non-singular case, also for the Laplacian operator.

Some results on boundary value problems for Choquard equations

Chiara Bernardini - Università di Trento

Abstract

We study the following nonlinear Choquard equation

$$-\Delta u + Vu = (I_\alpha * |u|^p)|u|^{p-2}u \qquad \text{in } \Omega \subset \mathbb{R}^N,$$

where $N \geq 2$, $p \in (1, +\infty)$ and $V(x)$ is a continuous radial function such that $\inf_{x\in\Omega} V > 0$.

First, assuming to have Neumann or Dirichlet boundary conditions, we prove existence of a positive radial solution when Ω is a ring-shaped domain or an exterior domain of the form $\mathbb{R}^N \setminus \bar{B}_a(0)$. We also provide a nonexistence result: if $p \geq \frac{N+\alpha}{N-2}$ the corresponding Dirichlet problem does not have any nontrivial regular solution in strictly star-shaped domains. Finally, when considering annular domains, letting $\alpha \to 0^+$ we obtain an existence result for the corresponding *local* problem with power-type nonlinearity.
This talk is based on a joint work with A. Cesaroni.

A variational analysis of nematic axisymmetric films

Giulia Bevilacqua - Università di Pisa

Abstract

Nematic films are thin fluid structures, ideally two dimensional, endowed with an in-plane degenerate nematic order. Some variational models for nematic films have been introduced by Giomi in 2012 and by Napoli and Vergori in 2018. At equilibrium, the shape of the nematic film results from the competition between surface tension, which favors the minimization of the area, and the nematic elasticity, which instead promotes the alignment of the molecules along a common direction. The main difference between the two mentioned approaches is the way to compute the surface derivative of the nematic vector field. In this seminar, I will briefly describe the two physical models and I will present a complete variational analysis of the model proposed by Giomi for revolution surfaces spanning two coaxial rings. This is a joint work with Chiara Lonati, Luca Lussardi and Alfredo Marzocchi.

Regularity in problems with high-codimensional weights

Gabriele Cora - Università di Torino

Abstract

In this talk, we present some recent results on the regularity of solutions to partial differential equations in divergence form, involving singular, degenerate, or superdegenerate weights of high codimension. These equations arise in fields such as harmonic map theory and general relativity.

Our primary focus will be on the model equation

$$-\mathrm{div}(|y|^a \nabla u(x,y)) = 0,$$

where $(x,y) \in \mathbb{R}^{N-n} \times \mathbb{R}^n$ with $n \geq 2$ and $a+n > 0$. Nevertheless, our methods extend to more general cases. Under suitable assumptions, we establish $C^{0,\alpha}$ and $C^{1,\alpha}$ regularity for solutions. Our approach combines techniques such as approximation via perforated domains, blow-up analysis, and a Liouville-type theorem.

This is a joint work with G. Fioravanti (Università di Torino) and S. Vita (Università di Pavia).

Layered patterns in reaction-diffusion models with Perona-Malik diffusions

Alessandra De Luca - Università di Torino

Abstract

In my talk I will present a recent result in collaboration with Marta Strani and Raffaele Folino involving a reaction-diffusion equation in a bounded interval of the real line, in the presence of a nonlinear diffusion of Perona-Malik type and a balanced bistable reaction term governed by a parameter $\vartheta > 1$. More specifically, after a brief overview on both linear and nonlinear models already available in literature, I will show the persistence of solutions with a transition layer structure in dependence on ϑ, with a particular emphasis on the critical case (that is when $\vartheta = 2$) where the so-called *metastable* phenomenon occurs.

Spectral stability for the Neumann Laplacian in domains with small holes

Lorenzo Liverani - FAU Erlangen

Abstract

In this talk, I will discuss the behavior of the spectrum of the Neumann Laplacian in domains with little holes excised from the interior. More precisely, I will investigate how the (single) eigenvalues of the Neumann Laplacian change under singular perturbations of the domain, obtained by removing Lipschitz sets which are "small" in a suitable sense. The main results are twofold: first, I will show a general, qualitative theorem, concerning the convergence of the eigenvalues to the ones of the unperturbed problem. The eigenvalue variation turns out to depend on a geometric quantity resembling the notion of (boundary) torsional rigidity. Next, in the particular case of a hole shrinking to a point, I will provide quantitative estimates for the rate of convergence, through a blow-up analysis. This presentation is based on a joint work with V. Felli and R. Ognibene.

On Cartan-Hadamard manifolds with very negative curvature

Ludovico Marini - Fukuoka University

Abstract

In this talk, we consider Cartan–Hadamard manifolds whose Ricci curvature grows polynomially at $-\infty$. We prove that several functional properties, which typically hold on manifolds with bounded curvature, remain true in this setting. For instance, density properties for Sobolev spaces and Calderón–Zygmund inequalities. The proof relies on the construction, via comparisons, of cutoff functions whose Hessian grows polynomially at infinity; and on a new class of second order Hardy-type inequalities. This is a joint work with Giona Veronelli.

Two cross-diffusion systems of Poisson–Nernst–Planck type for charge transport in ion channels: Analytical and Numerical challenges

Annamaria Massimini - TU Wien, ESILV

Abstract

Modeling concentrated ion mixtures in solvents like water is a complex research area with key applications in physiology (e.g., ion transport through protein channels) and electrochemistry (e.g., batteries). A crucial aspect is that the solvent actively transports ions, influencing their motion. Thus, when modeling ion fluxes, a solvent-induced drift term must be included. In dilute solutions, this drift depends linearly on the solvent's velocity and is typically considered independent of ion movement. In concentrated solutions or confined spaces like membranes or nanopores, where ion size is comparable to the channel diameter, ions occupy significant volume, leading to a saturated mixture. The solvent's volume fraction is determined by the ion volumes, and ionic fluxes depend on the concentration gradients of other species (cross-diffusion effects). Additionally, an electrical gradient across the membrane drives ion diffusion even after the concentration gradient reaches zero.

In this talk, I will present two models [1, 2] describing the evolution of ion mixtures in a polar solvent, governed by modified Poisson-Nernst-Planck equations with mixed boundary conditions. These equations include a drift term for electrostatic interactions, expressed through the electric potential gradient, coupled to concentrations via a Poisson-type equation. The saturated mixture gives rise to cross-diffusion effects, leading to non-diagonal and non-symmetric diffusion matrices, and resulting in strongly nonlinear and coupled PDEs. Analyzing these systems, called *cross-diffusion systems*, is challenging both analytically and numerically. I will overview the *boundedness-by-entropy* methods for establishing the existence of global-in-time bounded weak solutions and discuss uniqueness using the weak-strong uniqueness technique. Lastly, I will address challenges in developing numerical schemes, focusing on two finite volume approaches within exponentially adapted frameworks.

References

[1] Cancès, C., Herda, M., Massimini, A. (2023). Finite volumes for a generalized Poisson-Nernst-Planck system with cross-diffusion and size exclusion.

In *International Conference on Finite Volumes for Complex Applications* (pp. 57-73). Cham: Springer Nature Switzerland.

[2] Jüngel, A., Massimini, A. (2024). Analysis of a Poisson–Nernst–Planck–Fermi system for charge transport in ion channels. *Journal of Differential Equations*, 395, 38–68.

Non-existence theorems for solutions the Lane-Emden equation

Troy Petitt - Università di Milano

Abstract

In this talk, based on a joint work with Louis Dupaigne (Université Lyon 1 Claude Bernard) and Alberto Farina (Université de Picardie Jules Verne), we discuss non-existence results for stable solutions of the Lane-Emden equation

$$-\Delta u = u^p \,,$$

(for $p > 1$) in the half-space and general cones with homogeneous Dirichlet boundary conditions. Singular solutions, counterexamples, and some open problems are also discussed.

A case for the logarithmic Laplacian

Luigi Pollastro - Università di Torino

Abstract

In the past few decades the fractional Laplacian $(-\Delta)^s$ has capitalized the attention of the PDE community as an integrodifferential operator which is the nonlocal counterpart to the classical Laplacian.

Gaining popularity thanks to the seminal work of L. Caffarelli & L. Silvestre *"An extension problem related to the fractional Laplacian"*, it sees a widespread use thanks to its ability to model phenomena in which long term interactions between objects occur, leading to applications in particle physics, finance and population dynamics among others.

An interesting line of research is to investigate what happens to the operator as the index s approaches 0. It is in this instance that the logarithmic Laplacian L_Δ pops up, as the first order term in the Taylor expansion of the fractional Laplacian as s goes to 0.

The goal of this seminar is to present this fairly new operator, showcasing its basic features together with some open questions. Lastly, I will present symmetry results for an overdetermined and a rigidity problem involving the logarithmic Laplacian recently obtained in collaboration with N. Soave.

Symmetry breaking and rigidity results for semilinear elliptic equations

Camilla Polvara - Università di Milano

Abstract

We consider semilinear elliptic equations in spherical sectors with mixed boundary type conditions and in unbounded cones with Neumann boundary conditions.

The aim of the talk is to show that a radial symmetry result of Gidas-Ni-Nirenberg type for positive solutions does not hold in general when the cone is nonconvex, while a rigidity result holds in some class of non convex cones.

This symmetry breaking result is achieved by studying the Morse index of radial positive solutions and analyzing how it depends on the domain D on the unit sphere which spans the cone. In particular, we show that the Neumann eigenvalues of the Laplace Beltrami operator on D play a fundamental role in computing the Morse index.

Moreover, in the critical case, we show that a careful analysis related to the first Neumann eigenvalue of the Laplace Beltrami operator on D plays a crucial role in proof of the rigidity problem.

These results are obtained in a joint work with G. Ciraolo and F. Pacella.

References

[1] G. Ciraolo, F. Pacella, C. Polvara, *Symmetry breaking and instability for semilinear elliptic equations in spherical sectors and cones*, (2023), arXiv:2305.10176.

A capillarity one-phase Bernoulli free boundary problem

Giorgio Tortone - Università di Pisa

Abstract

This seminar explores one-phase Bernoulli free boundary problems within smooth containers, focusing on scenarios where the normal derivative of solutions is prescribed along the containers boundary. We examine the regularity of the free boundary and the structure of the wetting region, characterized by its contact with the fixed boundary.
Key findings include characterizing the contact angle in terms of container permeability and unveiling the smooth $(d-2)$-dimensional nature of the boundary of the wetting region, except for a possible closed set of dimension at most $d-5$.

This is based on a joint work with L. Ferreri and B. Velichkov.