# *(EXCERPTS OF THE )PROPOSAL FOR A PESC-PROGRAMME IN:* GLOBAL AND GEOMETRIC ASPECTS OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

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**SUMMARY**

The proposed PESC-Programme aims at the study of global and geometric properties of solutions of nonlinear partial differential equations (PDEs), from the view point of theory and applications.
Many problems in physics, medicine, finance and industry can be described by nonlinear partial differential equations. Their investigation has, in its own turn, become an independent field with many research directions. One of these, which this proposal is based on, is the analysis of geometric and global aspects of their solutions.
Particular problems, which will be considered by this Programme, are optimal transportation problems, free boundary problems, nonlinear diffusive systems, singular perturbations, nonlinear stochastic PDEs, regularity issues and global solutions of PDEs.

All the members of the present group have years of experience in PDE-research, complemented by their other research expertises (in abstract analysis, modelling with differential equations, numerical analysis). Joining the individual scientists of this group in a coherent activity and a coordination of the European mathematicians working within the proposed subject will definitely have a strong positive impact for

- Solution of open questions,
- Formulation of new problems and opening up of new research directions,
- Stimulation of interdisciplinary contacts,
- Training of students and junior researchers.

**Key Words:** Optimal transportation of mass, fully nonlinear PDEs, nonlinear evolutions, free boundary problems, singular perturbations.

## LETTER OF INTENT

*On behalf of the steering committee, we propose a PESC-programme in "Global and geometric aspects of nonlinear PDEs " for a period of five years. This proposal requests funding in order to bring together specialists from research groups in European countries, as well as Argentina, Chile, Japan, Russia, South Korea and US. The non-*
*EU partners will apply for outside funding to secure their participation costs. The European countries directly involved are: Austria, France, Germany, Greece, Italy, He Netherlands, Norway, Portugal, Spain, Sweden and Switzerland. We intend to organize workshops and conferences as well as coordinate and support exchanges between the different groups involved in the project. Training of young scientists, also in Summer/Winter schools, will be an important issue on our agenda.*

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# Status of the Topic, Goals and envisaged Achievements

The investigation of global and geometric properties of solutions of partial differential equations is a key research area in mathematics, which has many practical applications. Among them are problems appearing in fluid dynamics, flame propagation, semiconductors, nanotechnology, quantum mechanics etc..
Here we present some of the problems we intend to investigate within the frame of the suggested programme:

### 1) Optimal Transportation Problems

The problem of optimal transportation of mass has been much in focus in recent years, due to the large number of applications to nonlinear PDEs, fluid mechanics, shape optimization, geometric and functional inequalities, dynamical systems, Aubry-Mather theory and viscosity solutions of Hamilton-Jacobi equations, statistical mechanics, probability, economics. The problem, stated by Monge in 1781 (and later generalized by Kantorovich in 1942) can be stated as follows:
given two equi-mass distributions of a material described by functions f(x) and g(y) (e.g. an embankment and an excavation), find a transport map y=y(x) which carries the first distribution into the second and minimizes the transportation cost.
Generalizing, one can replace the distribution functions by positive measures of equal mass and replace the Euclidean distance by a more general cost function and again seek an optimal transportation map. This problem appears in many fields, including partial differential equations, fluid mechanics, shape optimization, and probability theory. The geometric and qualitative aspects of the problem (e.g. existence, uniqueness and characterization of the optimal transport map) have been studied by many authors and, as we mentioned, there is an impressive number of applications: nonlinear PDEs, shape optimization, statistics and statistical mechanics to quote just a few examples.
There is no doubt that this problem is in the mainstream of theoretical and applied mathematical research. In the last few years there have been several workshops/conferences devoted to this subject and more are to come.

### 2) Theory of nonlinear diffusion equations

We shall deal with the analysis of nonlinear parabolic equations, which arise in areas like fluid filtration, mass diffusion, particle transport and heat propagation. The degenerate or singular

character of these equations often causes free boundaries to appear. Ongoing work is oriented to new fields of applications like image processing.
*Topic 1: Self-similarity and asymptotic methods.* We shall study the existence of solutions which are invariant under transformation groups, a classical subject in continuum mechanics. New emphasis will be put on properties of stability and attraction, cf. 3).
*Topic 2: Problems in Combustion Theory, blow-up and extinction.* We have two main lines of interest: i) the explanation and calculation of phenomena of complete blow-up; ii) the theory of blow-up for systems.
*Topic 3: Higher-order equations.* The basic model is the thin film equation. We shall work on variational ineqalities, obstacle problems and singularity formation. New applications for these equation can be found in quantum mechanics and nanotechnology.
*Topic 4: Problems with strong convection.* Equations of this type arise for example in models of multiple transitions of phase as in water-ice-vapor, in biomathematics like in chemotaxis, in the flow of a fluid through a porous medium and in semiconductor devices. We shall study questions related to uniqueness, regularity and asymptotic behaviour including (uniform) boundedness and Harnack estimates.

**3) Entropy Methods for Nonlinear Diffusion and Kinetic Equations**

In the recent years a new technique has been developed for the analysis of the longtime behaviour of diffusive systems. This technique, now known under the name entropy-entropy dissipation method, is based on Boltzmann´s original work in gas-kinetics, namely on finding a time-decaying functional of the solution of the diffusive system under consideration, satisfying appropriate functional properties (like convexity) and deriving an equation for it in terms of its dissipation. In particular, the time-decay of the (relative) entropy can be determined when an estimate of the entropy dissipation in term of the (relative) entropy is known. Recently, this technique has led to exciting new results for Boltzmann equations, the Landau equation, quantum kinetic models, linear and nonlinear Fokker-Planck type systems. An important spin-off has been the proof of new Sobolev type inequalities.

We plan to apply the entropy-entropy dissipation technique to more complicated PDEs, like inhomogeneous (classical and quantum) kinetic systems, energy transport equations, Fokker-Planck systems with non-conservative drift etc.
We remark that the topics 1), 2) and 3) are of course deeply related with great opportunities of cross-fertilization and synergy.

**4) Singular Perturbations and Homogenisation of Nonlinear PDEs**

Many physical systems can be modelled as nonlinear PDEs, which depend on a small parameter which multiplies a highest order derivative (like the viscosity in gas/fluid-flows, the Debye length in ultra - integrated semiconductors etc.).Typically, the solutions of these PDEs exhibit singular behaviour as this parameter tends to zero. Problems of this type, which were for the first time studied by Prandtl in fluid-dynamics boundary layer theory, are called singular perturbation problems.
Very often, an efficient numerical simulation of these problems, and a qualitative (physical) understanding of these PDEs is impossible without deep mathematical analysis and qualitative information on solutions.

Typically, singular perturbation of nonlinear PDE leads to complicated boundary layer behaviour, to the occurrence of free boundaries and to singularities in physical space and/or time. Moreover, singular perturbation of kinetic equations gives rise to diffusive equations in position space. The following topics shall be investigated:

1) singular perturbations in flame propagation and in systems with dominant convection.

2) small - Debye length limits in semiconductor energy - transport and drift diffusion models
3) mean-free-path limits of kinetic equations.

Homogenisation problems are characterized by small wavelength oscillations of the (coefficients of the) involved differential operators. The difficulty is to find approximate simpler problems, whose solutions exhibit the correct macroscopic scale behaviour without having the need to accurately resolve the fine microscopic structure. Important homogenization problems occur in geophysics, quantum transport in crystals, electromagnetic fields in periodic media, etc. We remark that very often there are close connections to transport problems in random media, which connects this topic closely to Topic 8 of this proposal.
We expect progress in homogenization problems of quantum physics, connecting to the (already well developed) theory of homogenization of Hamilton Jacobi equations by means of viscosity solutions. Other problems to be treated include the homogenization of the (quadratic) energy density of linear and weakly nonlinear double scale PDEs by means of Wigner transforms. Also we shall analyse homogenization problems in kinetic theory and kinetic equations in random media.

**5) Regularity of unknown sets**

Regularity questions of unknown sets have been in focus for several decades. Many times, modeling phenomena in nature by partial differential equations give rise to unknown boundaries. It is of great importance to analyze these boundaries form a rigorous mathematical point of view.
Recent years have also seen many new directions as well as new techniques entering into this very technical field. Several areas such as potential theory, micromagnetics, image processing, computer vision, geometric measure theory, free boundary problems in physics/mechanics, and many other subjects have gained or regained from the new techniques.
The subject is growing at a fast pace and experts predict the resolution of several important unanswered questions within the next 5-10 years. A famous problem of this kind is the regularity of the free boundary for a minimization problem which leads to the so-called Bernoulli type free boundary condition, and has been subject for investigation in the last two decades. Partial results of this problem state the regularity of the free boundary at flat points. In two dimensions H.W. Alt and L.A. Caffarelli proved (1981) that the free boundary is regular. A recent result of Caffarelli, Jerrison and Kenig generalizes this to three dimensions. These results are relevant in the study of the stability and continuous dependence of free boundaries with respect to the data and have interesting applications in several applied problems and their numerical computations.

**6) Global Solutions in PDE/FBP**

The local study of solutions of PDEs is often carried out through a technique of "zoom" or blow-up analysis. This procedure reduces the problem to the classification of global solutions (solutions in the whole space). It is often employed, for instance, for minimal surfaces, free boundary problems and for semilinear equations.

In this direction, a conjecture posed by E. de Giorgi in 1978 has attracted a lot of attention lately, due to important recent progress made by several groups. The problem concerns semilinear elliptic equations and their global or entire bounded solutions which are monotone in one direction. The global problem is obtained after blowing up an interphase or phase transition between two different physical or chemical states, and it can be considered a perturbed version of the classificaction of entire minimal graphs. The conjecture is that their level sets are hyperplanes. This has been proven to be true in two dimensions by Ghoussoub and Gui, and in three dimensions by Ambrosio and Cabrè. The proofs use important techniques developed by Berestycki-Caffarelli-Nirenberg in their works on symmetry of solutions. The conjecture remains open in dimensions larger than three, but it is believed to be true at least up to eight dimensions, as in minimal graphs.

**7) Fully Nonlinear PDEs**

Fully nonlinear elliptic equations of the form F(D^2u,Du,u,x)=0 (i.e. nonlinear in the second derivatives) have important applications to many areas. In global differential geometry the prototype is the Monge-Ampere equation det (D^2 u)=f(x). The study of global problems concerning hypersurfaces of positive constant Gauss curvature (and the corresponding classification of global solutions) are among the current interests. The connections between affine differential geometry and k-Hessian equations (that is the elementary symmetric functions of the eigenvalues of D^2 u or of the principal curvatures) is also a topic under intense current research. A second field of great activity in fully nonlinear equations has arisen recently, following new results and applications of Monge-Ampere equations to problems of optimal transport maps (or allocation problems) and of optimal design. This is work developed by Caffarelli and Trudinger, among others.

Finally, the study of fully nonlinear equations in periodic media and their homogenizations, as well as connections with stochastic Aubry-Mather theory and eventually quantum mechanics are also of current interest.

**8) Nonlinear Stochastic PDEs**

There are several connections between PDEs, SPDEs and finance. We mention some of them:

1)PDEs and finance.

The theory of optimal stopping is central in economics. For example,

(i) the problem of finding the optimal time of starting (or stopping) an economic activity under uncertainty can often be formulated as an optimal stopping problem.

(ii) Second, it has been proved that the price of an American option in finance is given by the solution of an optimal stopping problem.

Today it is well known that an optimal stopping problem for a diffusion is equivalent to a free boundary problem, and such problems are again equivalent to certain variational inequaly involving partial differential operators (the generators of the underlying diffusions). We need to solve the corresponding PDEs in the so-called continuation region and combine this with high contact (smoothness) conditions. This gives an explicit link between PDEs and optimal stopping and hence with finance.

(ii) Stochastic control problems are also central in economics. For example. the problem of finding the optimal consumption rate and optimal portfolio for a trader in a financial market can be formulated as such a problem. The dynamic programming method gives a direct link between the problem of stochastic control of a diffusion and the nonlinear partial differential equation called the Hamilton-Jacobi-Bellman (HJB) - equation, which involves the generator of the diffusion.

2) SPDEs and finance.

(i) The Musiela equation is an SPDE which occurs in the theory of interest rates in finance.

(ii) In order to find the best estimate at a given time of a stochastic system based on noisy observations of the system, filtering theory is central. In the linear case the famous Kalman filter solves this problem explicitly. In the nonlinear case the solution is described in terms of an SPDE called the Zakai equation.

(iii) If one wants to study the problem of optimal control of a stochastic system with partial information (i.e. where some of the parameters in the system are not known exactly, only partially through noisy observations), one arrives at the Zakai equation (ii) and is led to the study of optimal control of SPDEs. This is a challenging area of research where little is known so far.

**9) Applications of Free Boundary Problems**
We propose to study some specific models related to filtration of fluids in porous media. In particular we will consider:

1) Gravity segregation in porous media. The aim is to study the segregation of foam in horizontal reservoirs. This leads to a three-phase elliptic free boundary problem with free boundaries separating the regions containing foam, gas and water. In particular the behaviour of the three free boundaries near the triple point (where complete segregation takes place) poses new and challenging questions.

2) Crystal dissolution and precipitation. This involves a model for the convective-dispersive transport of solutes in a porous medium undergoing precipitation/dissolution reactions with respect to the solid matrix. In mathematical terms one needs to consider two transport equations and a set-valued first order reaction equation. A previous travelling wave analysis indicates the conditions for which dissolution/precipitation fronts (free boundaries)occur. Mathematical issues are the regularity and qualitative properties of the free boundaries and the stabilization of profiles towards travelling waves.

## Collateral Activities

The programme will broaden in incorporate other activities, which are already planned for the next 4-5 years .

1) The Nonlinear Analysis Conference at the Wolfgang Pauli-Institute Vienna in May 2002, organized by two of the principal proposers, was the starting point for this proposal. There are plans for a successive conference in two years in an as of yet undecided location.

2) A workshop in Stockholm in June 2003 with the same title as the proposed programme will take place. The initial concrete planning for this ESF-Programme could be undertaken there.

3) An International Conference on "Free Boundary Problems: Theory and Applications is foreseen to be held in June 2005 in Portugal. The participation of this ESF-Programme would enhance the European research in fundamental mathematics relevant to that important interdisciplinary field.

4) There is a plan to apply for a Mittag-Leffler Programme year in 2007/2008 in Stockholm. The organising committee for the M-L application consists of: Björn Engquist, Björn Gustafsson, Henrik Shahgholian, and Anders Szepessy. Also several participants of this Programme will take part. We have already discussed the matter with the Director of M-L institute professor K.O. Widman and he has strongly encouraged us to apply for a program in nonlinear PDE.

## Programme work plan

1) Short scientific visits between members of the research groups: an average of 2 members/group and year, for a period of 1-2 weeks.

2) Collaborative research projects for young researchers: 2-3 visits of Ph.D. students and 1-2 visits of postdoctoral researchers; up to six months all together. Also joint supervision of Ph.D. students by two senior researchers of this network are planned for.

Planed Conferences (subject to change)

*2004:* Global and geometric aspects in mass transfer problems, and related topics
*2005*: Global solutions in pde/fbp and their applications
*2006:* PDE and FBP in industrial applications
*2007:* Nonlinear diffusion problems
*2008:* The past the future of global and geometric aspects in PDE

Summer/Winterschools

Winter school in *Global methods in* PDE (2005)
Winter school in *mass transfer problems (2007)*
Winter/Summer school in *Qualitative* PDE; (2008)

## European Dimension

This programme strives to coordinate and extend efforts by European scientists in an important area of mathematics. Researchers of twelve European countries form the steering committee and will participate in the scientific activities of the programme.
In the past two decades the analysis of nonlinear partial differential equations has been increasingly dominated by scientific centres outside Europe, particularly in the USA, although a significant pool of knowledge exists in European countries. We believe that our proposed ESF-Programme will create enough momentum through synergetic effects between the dispersed European research groups such that novel science will be created here. This will ultimately play a major role in persuading young European researchers (pre-and post-Docs) to continue their careers in Europe avoiding the brain - drain which has been going on in the area of nonlinear analysis over the last decades. Also we believe that this Programme will significantly contribute to attract highly talented students all over Europe and to develop the European human resources in this key research area in mathematics

## Related European Networks and Programmes

Research Training Networks in the 5th Framework programme of the EU
a) HYKE = "Hyperbolic and Kinetic Equations: Asymptotics, Numerics, Application, Coordinator N. Mauser (WPI, Vienna)
b) Nonlinear Partial Differential Equations describing Front Propagation and other Singular Phenomena, Coordinator M.Bertsch (Rome)

Our proposed ESF-Programme focuses strongly on the methodological base for nonlinear PDEs, particularly on global and geometric properties, while the above mentioned related networks deal with specific types of linear and nonlinear PDEs from a somewhat more applied point of view. We see a significant complimentarity of these projects, in particular we expect to provide methodology for these IHP networks and vice versa, our ESF-Programme will benefit from numerical techniques developed in HYKE as well as from new areas of applications which will be opened up by those two networks.
Other Programmes (like the ESF-Programme SPECT) focus entirely on linear PDEs, with little direct scientific relation to our proposed ESF-Programme.

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