ANALISI MATEMATICA I Seconda prova in itinere

3/2/2006

TEMA ....1....

1) $\lim_{x\to 0} f(x) = \frac{2}{9}$ ($\sqrt[3]{1+x^2} - 1 \underset{x\to 0}{\backsim} \frac{x^2}{3}$; $\log(1-x+2x^2) + x = \{(-x+2x^2) - \frac{1}{2}(-x+2x^2)^2 + o((-x+2x^2)^2)\} + x = \frac{3}{2}x^2 + o(x^2)$ per $x \to 0$)

2) $f(x) = \frac{\pi}{8} + (\frac{\pi}{4} + \frac{1}{2})(x - \frac{1}{2}) + \frac{1}{2}(x - \frac{1}{2})^2 + o\big((x - \frac{1}{2})^2\big)$ $x \to \frac{1}{2}$

($f'(x) = \arctan 2x + \frac{2x}{1+4x^2}$; $f''(x) = \frac{2}{1+4x^2} + \frac{-8x^2+2}{(1+4x^2)^2}$)

3) - continuita': $\forall a$, $b = -2$

- derivabilita': $a \in \{-2, 3\}$, $b = -2$

4) $f \in C^\infty(\mathbf{R}) \Longrightarrow$i) f e' invertibile su $\mathbf{R}$, perche' $f'(x) = 3e^{x^3-6x^2+12x}(x^2 - 4x + 4) \geq 0$ per ogni $x \in \mathbf{R}$ e $f'(x) = 0 \Longleftrightarrow x = 2$; ii) $f(\mathbf{R}) = (-3, +\infty)$; iii) $g'(-2) = \frac{1}{f'(g(-2))} = \frac{1}{f'(0)} = \frac{1}{12}$ ($f(0) = -2$, $g(-2) = 0$)

5) $\int \frac{1}{\sqrt[3]{x^2} - 4\sqrt[3]{x} + 3} dx = \left(\int \frac{3t^2}{t^2-4t+3} dt\right)_{t=\sqrt[3]{x}} = \left(\int (3 + \frac{12t-9}{t^2-4t+3}) dt\right)_{t=\sqrt[3]{x}} = \left(\int (3 + \frac{27}{2}\frac{1}{t-3} - \frac{3}{2}\frac{1}{t-1}) dt\right)_{t=\sqrt[3]{x}} = \big(3t + \frac{27}{2}\log|t-3| - \frac{3}{2}\log|t-1|\big)\big)_{t=\sqrt[3]{x}} + c =$

$3\sqrt[3]{x} + \frac{27}{2}\log|\sqrt[3]{x} - 3| - \frac{3}{2}\log|\sqrt[3]{x} - 1|) + c$ $c \in \mathbf{R}$

6) - Insieme di definizione: $E_F = (\frac{1}{2}, +\infty)$ ($f(t) = \dfrac{t^2 - 4t + 3}{(2t-1)^3}$ e' continua in $E_f = \mathbf{R} - \{\frac{1}{2}\}$; per $t \to \frac{1}{2}$ $f(t) \asymp \frac{1}{(2t-1)^3}$ non integrabile in senso improprio in $U(\frac{1}{2})$; estremo inferiore di integrazione 1). $F \in C(E_F)$.

- Limiti alla frontiera, eventuali asintoti:

$\lim_{x\to(\frac{1}{2})^+} F(x) = -\infty$ (per $t \to (\frac{1}{2})^+$ $f(t) \asymp \frac{1}{(2t-1)^3}$ non integrabile in senso improprio in $U(\frac{1}{2})) \Longrightarrow x = \frac{1}{2}$ asintoto verticale.

$\lim_{x\to+\infty} F(x) = +\infty$ (per $t \to +\infty$ $f(t) \backsim \frac{1}{8t}$ non integrabile in s.i. in $U(+\infty)$).

Non esiste asintoto obliquo per $x \to +\infty$, perche' $\lim_{x\to+\infty} \frac{F(x)}{x} \overset{H}{=} \lim_{x\to+\infty} F'(x) = 0^+$.

- $F(1) = 0$.

- Crescere e decrescere:

$F'(x) = \dfrac{x^2 - 4x + 3}{(2x-1)^3}$ $x > \frac{1}{2}$. $F'(x) > 0 \Longleftrightarrow \frac{1}{2} < x < 1$, $x > 3$ $\Longrightarrow$ $x = 1$ punto di massimo relativo e $x = 3$ punto di minimo relativo ($F(3) < 0$).

- Convessita' e concavita':

$$F''(x) = \frac{-2x^2 + 14x - 14}{(2x-1)^4} \qquad x > \tfrac{1}{2}$$

$F''(x) > 0 \iff \frac{7-\sqrt{21}}{2} < x < \frac{7+\sqrt{21}}{2} \implies x = \frac{7-\sqrt{21}}{2}$ e $x = \frac{7+\sqrt{21}}{2}$ sono punti di flesso.

- Grafico:

7) Esiste perche': i) $f \in C((0,+\infty))$; ii) per $x \to 0^+$ $f(x) \backsim \frac{9x^3}{x^2(\log 3)^3} \to 0$; iii) per $x \to +\infty$

$f(x) \backsim \frac{3x}{x^2(\log x)^3} = \frac{3}{x(\log x)^3}$ integrabile in senso improprio in $U(+\infty)$.