ABSTRACTS

Zoltan Balogh: **"Fractals in Carnot groups"**. We study the Hausdorff dimension of invariant sets of Iterated Function Systems on Carnot groups. We prove that generically for self-similar IFS the similarity dimension of the invariant set coincides with the Hausdorff dimension with respect to the Carnot-Caratheodory metric. We give quantitative estimates of the Hausdorff dimension of the exceptional set of translation parameters for which the Hausdorff dimension is less than the similarity dimension.

Andrea Calogero: **"Horizontal normal map and H–subdifferentiability in Carnot groups"**. We present some recent results obtained with R. Pini about the notion of H–subdifferential and H–normal map of a function u on Carnot groups $\mathbf{G}$, based on the sub–Riemannian structure. We show that some properties of the subdifferential in the Euclidean setting are inherited. In particular, a characterization of the convexity (the so called "weak horizontal convexity") of a function is given via the nonemptiness of the H–subdifferential $\partial_H u(g)$ at every point $g \in \mathbf{G}$.
When $\mathbf{G}$ is the Heisenberg group $\mathbb{H}$, we give a monotonicity result for the H–normal map $\partial_H u$ of a suitable strictly convex radial function u, and we suggest a definition of the Monge–Ampère measure of a function u via its H–normal map. Some of the results of this talk are available in http://arxiv.org/abs/0811.2277arXiv.

Luca Capogna: **"Regularity of certain minimal graphs in the subriemannian Heisenberg group"**. Minimal surfaces in the sub-Riemannian Heisenberg group can be constructed by means of a Riemannian approximation scheme, as limit of Riemannian minimal surfaces. We study the regularity of Lipschitz, non-characteristic minimal surfaces which arise as such limits. Our main results (joint with Citti and Manfredini) are a-priori estimates on the solutions of the approximating Riemannian PDE and the ensuing C^∞ regularity of the sub-Riemannian minimal surface along its Legendrian foliation.

Valentina Casarino: **"On the L^p-summability of Riesz means for the sublaplacian on complex spheres"**. We discuss the L^p-convergence of the Riesz means $S_R^\delta(f)$ for the sublaplacian on the sphere S^{2n-1} in the complex n-dimensional space $\mathbb{C}^n$. We show that $S_R^\delta(f)$ converges to f in $L^p(S^{2n-1})$ when $\delta > \delta(p) := (2n-1)|\frac{1}{2} - \frac{1}{p}|$. The index $\delta(p)$ improves the one found by Alexopoulos and Lohoué $2n|\frac{1}{2} - \frac{1}{p}|$, and it coincides with the index found by Mauceri and, with different methods, by Müller in the case of the sublaplacian on the Heisenberg group (joint work with Marco Peloso).

Michael Cowling: **"Mappings of groups that send cosets to cosets"**. We show that some mappings of Lie groups that send cosets to cosets are extended affine, that is, composed of translations, homomorphisms, and perhaps inversion.

Filippo De Mari: **"Rigidity of Carnot groups relative to multicontact structures"**. A multicontact structure on a Carnot group is a refinement of the classical contact structure. We give an answer to conjectures of M. Cowling and A. Korányi, proving that the group of the diffeomorphisms that preserve the multicontact structure is always finite dimensional. Our method relies on Tanaka prolongation theory (joint work with A. Ottazzi).

Roberto Monti: **"Regularity of H-convex sets"**. We discuss first and second order regularity properties of the boundary of H-convex sets in Carnot groups of step 2. We prove that the noncharacteristic part of the boundary has locally the intrinsic cone property and that it is foliated by intrinsic Lipschitz curves that are twice differentiable almost everywhere.

Raul Serapioni: **"Characterization of intrinsic rectifiable sets"**. Intrinsic rectifiable sets in Carnot groups are defined as countable unions of intrinsic Lipschitz graphs or as countable unions of subsets of intrinsic C^1 submanifolds. In a joint work together with Pertti Mattila and Francesco Serra Cassano it has been proved that, inside Heisenberg groups, it is possibile to characterize intrinsic rectifiable sets, with positive lower density, in terms of almost everywhere existence of approximate tangent subgroups or of tangent measures.

Jeremy Tyson: **"Lipschitz, Holder and Sobolev Peano cubes"**. Standard constructions of Peano curves yield surjections which are Holder continuous or differentiable a.e. Which metric spaces receive a continuous surjection from $[0,1]^n$ which is Lipschitz? Holder? Sobolev? differentiable a.e. (in the metric differential sense of Kirchheim)?

We show that every length compact metric space is the image of the n-dimensional cube (for each $n \geq 2$) by a continuous map which is metrically differentiable a.e. and lies in a suitably defined Sobolev class $W^{1,n}$. Moreover, every compact quasiconvex doubling metric space is the image of the n-dimensional unit cube under a Hölder continuous map, or under a Lipschitz continuous map if n is sufficiently large. The maps which we construct factor through Lipschitz maps from compact quasiconvex doubling $\mathbb{R}$-trees in Hilbert space. As an application, we verify that the first Heisenberg group is the image of $\mathbb{R}^5$ under a Lipschitz map. The existence of such a surjection contrasts strongly with the pure unrectifiability of the Heisenberg group in large dimensions. We also construct highly regular contact surjections between Carnot groups with maximally singular horizontal differential.

This is joint work with Piotr Hajłasz.

Ben Warhurst: **Prolongation theory and rigidity of Carnot groups** In this talk we introduce the prolongations in the usual sense of Singer-Sternberg and in the sense of Tanaka, and apply them to the rigidity problem. In particular, we will show how these techniques prove all the known results in a unified and perhaps easier fashion. As an example we demonstrate the rigidity of H-type algebras. In fact, our proof extends to a wider class of algebras which we call J-type.

More generally, we will discuss some characterizations of nonrigidity. Let $\mathfrak{h}$ be the subalgebra consisting of strata preserving derivations which vanish on all strata except the first. A Carnot group G is nonrigid if and only if one of the following equivalent conditions hold:

(i) The complexification of $\mathfrak{h}$ contains rank one elements.

(ii) In the complexification of the Lie algebra of G there exists a vector X in the first stratum such that $\mathrm{rank}(\mathrm{ad}_X) = 1$.

This is joint work with A. Ottazzi.