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*Instructions: fill **completely** the bubbles with the digits of the SID (one for each column); in the lower part of the sheet, fill **completely** the bubbles with the correct answers to the corresponding question. Use a black or dark blue pen or pencil, trying to fill completely the inside of the bubble. Write only in the designated areas.*

**Last Name**:.......................... **First name**:.......................... **Signature**:..........................

**Mark the answers of the multiple-choice questions**

**(1)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

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**(13)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

**(14)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

**(1)** Let $X$, $Y$, $Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Y$ implies $Z$. If we also know that $X$ is false, we can conclude that

- ☐ *(a)* $Y$ is false and $Z$ is false.
- ☐ *(b)* $Z$ implies $X$.
- ☐ *(c)* $Y$ is false.
- ☐ *(d)* $Y$ is false, $Z$ is false and $Z$ implies $X$.
- ☐ *(e)* $Z$ is false.
- ☐ *(f)* No conclusion can be drawn.
- ☐ *(g)* $Y$ is false and $Z$ implies $X$.

**(2)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "At least one of $X$ and $Y$ are true", we need to show that

- ☐ *(a)* $X$ does not imply $Y$, and $Y$ does not imply $X$.
- ☐ *(b)* $X$ is false.
- ☐ *(c)* Exactly one of $X$ and $Y$ are false.
- ☐ *(d)* $X$ is true if and only if $Y$ is false.
- ☐ *(e)* $X$ and $Y$ are both false.
- ☐ *(f)* At least one of $X$ and $Y$ are false.
- ☐ *(g)* $Y$ is false.

**(3)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "$X \implies Y$", we need to show that

- ☐ *(a)* $X$ is true, but $Y$ is false.
- ☐ *(b)* $Y$ is false.
- ☐ *(c)* $X$ is false.
- ☐ *(d)* Exactly one of $X$ and $Y$ are false.
- ☐ *(e)* At least one of $X$ and $Y$ is false.
- ☐ *(f)* $X$ and $Y$ are both false.
- ☐ *(g)* $Y$ is true, but $X$ is false.

**(4)** Let $P(n, m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that "For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is true", then we need to prove that

- ☐ *(a)* There exists an integer $m$ such that $P(n, m)$ is false for all integers $n$.
- ☐ *(b)* For every integer $m$, there exists an integer $n$ such that $P(n, m)$ is false.
- ☐ *(c)* There exists an integer $n$ such that $P(n, m)$ is false for all integers $m$.
- ☐ *(d)* There exists integers $n$,$m$ such that $P(n, m)$ is false.
- ☐ *(e)* For every integer $n$, and every integer $m$, the property $P(n, m)$ is false.
- ☐ *(f)* If $P(n, m)$ is true, then $n$ and $m$ are not integers.
- ☐ *(g)* For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is false.

**(5)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that "$P(x)$ is true for all $x$ of type $X$", then we have to

- ☐ *(a)* Assume there exists an $x$ of type $X$ for which $P(x)$ is true, and derive a contradiction.
- ☐ *(b)* Show that for every $x$ in $X$, $P(x)$ is false.
- ☐ *(c)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.
- ☐ *(d)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type $X$.
- ☐ *(e)* Show that there are no objects $x$ of type $X$.
- ☐ *(f)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(g)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.

**(6)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "Both $X$ and $Y$ are true", we need to show that

- ☐ *(a)* Exactly one of $X$ and $Y$ are false.
- ☐ *(b)* $X$ is true if and only if $Y$ is false.
- ☐ *(c)* $X$ is false.
- ☐ *(d)* $Y$ is false.
- ☐ *(e)* $X$ does not imply $Y$, and $Y$ does not imply $X$.
- ☐ *(f)* At least one of $X$ and $Y$ are false.
- ☐ *(g)* $X$ and $Y$ are both false.

**(7)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that "$P(x)$ is true for some $x$ of type $X$", then we have to

- ☐ *(a)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(b)* Assume that $P(x)$ is true for every $x$ in $X$, and derive a contradiction.
- ☐ *(c)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.
- ☐ *(d)* Show that there are no objects $x$ of type $X$.
- ☐ *(e)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.
- ☐ *(f)* Show that for every $x$ in $X$, $P(x)$ is false.
- ☐ *(g)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type $X$.

**(8)** Let $P(n,m)$ be a property about two integers $n$ and $m$. If we want to prove that "For every integer $n$, there exists an integer $m$ such that $P(n,m)$ is true", then we should do the following:

- ☐ *(a)* Let $m$ be an arbitrary integer. Then find an integer $n$ (possibly depending on $m$) such that $P(n,m)$ is true.
- ☐ *(b)* Let $n$ and $m$ be arbitrary integers. Then show that $P(n,m)$ is true.
- ☐ *(c)* Let $n$ be an arbitrary integer. Then find an integer $m$ (possibly depending on $n$) such that $P(n,m)$ is true.
- ☐ *(d)* Show that whenever $P(n,m)$ is true, then $n$ and $m$ are integers.
- ☐ *(e)* Find an integer $m$ such that $P(n,m)$ is true for every integer $n$.
- ☐ *(f)* Find an integer $n$ such that $P(n,m)$ is true for every integer $m$.
- ☐ *(g)* Find an integer $n$ and an integer $m$ such that $P(n,m)$ is true.

**(9)** Let $X$, $Y$, $Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Y$ implies $Z$. If we also know that $Y$ is false, we can conclude that

- ☐ *(a)* None of the above conclusions can be drawn.
- ☐ *(b)* $Z$ is false.
- ☐ *(c)* $Z$ is false and $X$ implies $Z$. Correct Answer. $X$ is false and $X$ implies $Z$.
- ☐ *(d)* $X$ is false.
- ☐ *(e)* $X$ implies $Z$.
- ☐ *(f)* $X$ is false and $Z$ is false and $X$ implies $Z$.

**(10)** Let $P(n,m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that "There exists an integer $n$ such that $P(n,m)$ is true for all integers $m$", then we need to prove that

- ☐ *(a)* There exists integers $n$,$m$ such that $P(n,m)$ is false.
- ☐ *(b)* There exists an integer $n$ such that $P(n,m)$ is false for all integers $m$.
- ☐ *(c)* For every integer $m$, there exists an integer $n$ such that $P(n,m)$ is false.
- ☐ *(d)* If $P(n,m)$ is true, then $n$ and $m$ are not integers.
- ☐ *(e)* For every integer $n$, there exists an integer $m$ such that $P(n,m)$ is false.
- ☐ *(f)* For every integer $n$, and every integer $m$, the property $P(n,m)$ is false.
- ☐ *(g)* There exists an integer $m$ such that $P(n,m)$ is false for all integers $n$.

**(11)** Suppose one wishes to prove that "if all $X$ are $Y$, then all $Z$ are $W$". To do this, it would suffice to show that

- ☐ *(a)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(b)* All $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(c)* All $X$ are $Z$, and all $W$ are $Y$.
- ☐ *(d)* All $Y$ are $Z$, and all $W$ are $X$.
- ☐ *(e)* All $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(f)* All $Y$ are $X$, and all $Z$ are $W$.
- ☐ *(g)* All $Z$ are $Y$, and all $X$ are $W$.

**(12)** Let $X$ and $Y$ be statements. If we know that $X$ implies $Y$, then we can also conclude that

- ☐ *(a)* At least one of $X$ and $Y$ is true.
- ☐ *(b)* $X$ cannot be false.
- ☐ *(c)* If $Y$ is true, then $X$ is true.
- ☐ *(d)* $Y$ cannot be false.
- ☐ *(e)* If $Y$ is false, then $X$ is false.
- ☐ *(f)* $X$ is true, and $Y$ is also true.
- ☐ *(g)* If $X$ is false, then $Y$ is false.

**(13)** Suppose one wishes to prove that "if some $X$ are $Y$, then some $Z$ are $W$". To do this, it would suffice to show that

- ☐ *(a)* Some $Z$ are $X$, and some $Y$ are $W$.
- ☐ *(b)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(c)* All $X$ are $Z$, and some $Y$ are $W$.
- ☐ *(d)* Some $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(e)* All $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(f)* Some $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(g)* All $X$ are $Z$, and all $Y$ are $W$.

**(14)** Let $X$ and $Y$ be statements. Which of the following strategies is NOT a valid way to show that "$X \implies Y$"?

- ☐ *(a)* Assume that $X$ is true, and $Y$ is false, and deduce a contradiction.
- ☐ *(b)* Show that either $X$ is false, or $Y$ is true, or both.
- ☐ *(c)* Show that $X$ implies some intermediate statement $Z$, and then show that $Z \implies Y$.
- ☐ *(d)* Show that some intermediate statement $Z \implies Y$, and then show that $X \implies Z$.
- ☐ *(e)* Assume that $X$ is false, and $Y$ is true, and deduce a contradiction.
- ☐ *(f)* Assume that $X$ is true, and then use this to show that $Y$ is true.
- ☐ *(g)* Assume that $Y$ is false, and then use this to show that $X$ is false.

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**Last Name**:........................ **First name**:........................ **Signature**:........................

**Mark the answers of the multiple-choice questions**

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**(1)** Let $X$, $Y$, $Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Y$ implies $Z$. If we also know that $Y$ is false, we can conclude that

- ☐ *(a)* $X$ is false and $Z$ is false and $X$ implies $Z$.
- ☐ *(b)* $Z$ is false and $X$ implies $Z$. Correct Answer. $X$ is false and $X$ implies $Z$.
- ☐ *(c)* $Z$ is false.
- ☐ *(d)* $X$ implies $Z$.
- ☐ *(e)* None of the above conclusions can be drawn.
- ☐ *(f)* $X$ is false.

**(2)** Suppose one wishes to prove that "if all $X$ are $Y$, then all $Z$ are $W$". To do this, it would suffice to show that

- ☐ *(a)* All $Y$ are $Z$, and all $W$ are $X$.
- ☐ *(b)* All $X$ are $Z$, and all $W$ are $Y$.
- ☐ *(c)* All $Y$ are $X$, and all $Z$ are $W$.
- ☐ *(d)* All $Z$ are $Y$, and all $X$ are $W$.
- ☐ *(e)* All $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(f)* All $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(g)* All $Z$ are $X$, and all $W$ are $Y$.

**(3)** Suppose one wishes to prove that "if some $X$ are $Y$, then some $Z$ are $W$". To do this, it would suffice to show that

- ☐ *(a)* Some $Z$ are $X$, and some $Y$ are $W$.
- ☐ *(b)* All $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(c)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(d)* All $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(e)* Some $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(f)* All $X$ are $Z$, and some $Y$ are $W$.
- ☐ *(g)* Some $Z$ are $X$, and all $Y$ are $W$.

**(4)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "$X \implies Y$", we need to show that

- ☐ *(a)* $X$ is false.
- ☐ *(b)* At least one of $X$ and $Y$ is false.
- ☐ *(c)* $X$ and $Y$ are both false.
- ☐ *(d)* $Y$ is false.
- ☐ *(e)* $X$ is true, but $Y$ is false.
- ☐ *(f)* Exactly one of $X$ and $Y$ are false.
- ☐ *(g)* $Y$ is true, but $X$ is false.

**(5)** Let $X$ and $Y$ be statements. Which of the following strategies is NOT a valid way to show that "$X \implies Y$"?

- ☐ *(a)* Show that some intermediate statement $Z \implies Y$, and then show that $X \implies Z$.
- ☐ *(b)* Assume that $X$ is true, and $Y$ is false, and deduce a contradiction.
- ☐ *(c)* Show that either $X$ is false, or $Y$ is true, or both.
- ☐ *(d)* Assume that $Y$ is false, and then use this to show that $X$ is false.
- ☐ *(e)* Assume that $X$ is false, and $Y$ is true, and deduce a contradiction.
- ☐ *(f)* Assume that $X$ is true, and then use this to show that $Y$ is true.
- ☐ *(g)* Show that $X$ implies some intermediate statement $Z$, and then show that $Z \implies Y$.

**(6)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that "$P(x)$ is true for all $x$ of type $X$", then we have to

- ☐ *(a)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.
- ☐ *(b)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type $X$.
- ☐ *(c)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.
- ☐ *(d)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(e)* Assume there exists an $x$ of type $X$ for which $P(x)$ is true, and derive a contradiction.
- ☐ *(f)* Show that for every $x$ in $X$, $P(x)$ is false.
- ☐ *(g)* Show that there are no objects $x$ of type $X$.

**(7)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "At least one of $X$ and $Y$ are true", we need to show that

- ☐ *(a)* $X$ and $Y$ are both false.
- ☐ *(b)* $X$ is true if and only if $Y$ is false.
- ☐ *(c)* Exactly one of $X$ and $Y$ are false.
- ☐ *(d)* $X$ does not imply $Y$, and $Y$ does not imply $X$.
- ☐ *(e)* $X$ is false.
- ☐ *(f)* $Y$ is false.
- ☐ *(g)* At least one of $X$ and $Y$ are false.

**(8)** Let $X, Y, Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Y$ implies $Z$. If we also know that $X$ is false, we can conclude that

- ☐ *(a)* $Y$ is false, $Z$ is false and $Z$ implies $X$.
- ☐ *(b)* $Y$ is false and $Z$ implies $X$.
- ☐ *(c)* $Z$ is false.
- ☐ *(d)* $Z$ implies $X$.
- ☐ *(e)* No conclusion can be drawn.
- ☐ *(f)* $Y$ is false and $Z$ is false.
- ☐ *(g)* $Y$ is false.

**(9)** Let $P(n, m)$ be a property about two integers $n$ and $m$. If we want to prove that "For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is true", then we should do the following:

- ☐ *(a)* Show that whenever $P(n, m)$ is true, then $n$ and $m$ are integers.
- ☐ *(b)* Find an integer $n$ and an integer $m$ such that $P(n, m)$ is true.
- ☐ *(c)* Let $m$ be an arbitrary integer. Then find an integer $n$ (possibly depending on $m$) such that $P(n, m)$ is true.
- ☐ *(d)* Find an integer $m$ such that $P(n, m)$ is true for every integer $n$.
- ☐ *(e)* Let $n$ and $m$ be arbitrary integers. Then show that $P(n, m)$ is true.
- ☐ *(f)* Let $n$ be an arbitrary integer. Then find an integer $m$ (possibly depending on $n$) such that $P(n, m)$ is true.
- ☐ *(g)* Find an integer $n$ such that $P(n, m)$ is true for every integer $m$.

**(10)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that "$P(x)$ is true for some $x$ of type $X$", then we have to

- ☐ *(a)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(b)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.
- ☐ *(c)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type $X$.
- ☐ *(d)* Show that for every $x$ in $X$, $P(x)$ is false.
- ☐ *(e)* Assume that $P(x)$ is true for every $x$ in $X$, and derive a contradiction.
- ☐ *(f)* Show that there are no objects $x$ of type $X$.
- ☐ *(g)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.

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- ☐ *(a)* For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is false.
- ☐ *(b)* There exists integers $n$,$m$ such that $P(n, m)$ is false.
- ☐ *(c)* If $P(n, m)$ is true, then $n$ and $m$ are not integers.
- ☐ *(d)* There exists an integer $n$ such that $P(n, m)$ is false for all integers $m$.
- ☐ *(e)* There exists an integer $m$ such that $P(n, m)$ is false for all integers $n$.
- ☐ *(f)* For every integer $m$, there exists an integer $n$ such that $P(n, m)$ is false.
- ☐ *(g)* For every integer $n$, and every integer $m$, the property $P(n, m)$ is false.

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- ☐ *(c)* $X$ is true if and only if $Y$ is false.
- ☐ *(d)* $X$ and $Y$ are both false.
- ☐ *(e)* $X$ does not imply $Y$, and $Y$ does not imply $X$.
- ☐ *(f)* $X$ is false.
- ☐ *(g)* At least one of $X$ and $Y$ are false.

**(13)** Let $X$ and $Y$ be statements. If we know that $X$ implies $Y$, then we can also conclude that

- ☐ *(a)* If $Y$ is false, then $X$ is false.
- ☐ *(b)* $X$ cannot be false.
- ☐ *(c)* $X$ is true, and $Y$ is also true.
- ☐ *(d)* If $Y$ is true, then $X$ is true.
- ☐ *(e)* $Y$ cannot be false.
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- ☐ *(g)* If $X$ is false, then $Y$ is false.

**(14)** Let $P(n,m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that "There exists an integer $n$ such that $P(n,m)$ is true for all integers $m$", then we need to prove that

- ☐ *(a)* There exists integers $n$,$m$ such that $P(n,m)$ is false.
- ☐ *(b)* For every integer $n$, there exists an integer $m$ such that $P(n,m)$ is false.
- ☐ *(c)* For every integer $n$, and every integer $m$, the property $P(n,m)$ is false.
- ☐ *(d)* For every integer $m$, there exists an integer $n$ such that $P(n,m)$ is false.
- ☐ *(e)* There exists an integer $n$ such that $P(n,m)$ is false for all integers $m$.
- ☐ *(f)* There exists an integer $m$ such that $P(n,m)$ is false for all integers $n$.
- ☐ *(g)* If $P(n,m)$ is true, then $n$ and $m$ are not integers.

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**Last Name**:. . . . . . . . . . . . . . . . . . . . . . . . **First name**:. . . . . . . . . . . . . . . . . . . . . . . . **Signature**:. . . . . . . . . . . . . . . . . . . . . . . . .

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**Mark the answers of the multiple-choice questions**

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**(14)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

**(1)** Let $X$ and $Y$ be statements. Which of the following strategies is NOT a valid way to show that "$X \implies Y$"?

☐ *(a)* Assume that $Y$ is false, and then use this to show that $X$ is false.

☐ *(b)* Assume that $X$ is false, and $Y$ is true, and deduce a contradiction.

☐ *(c)* Show that some intermediate statement $Z \implies Y$, and then show that $X \implies Z$.

☐ *(d)* Assume that $X$ is true, and $Y$ is false, and deduce a contradiction.

☐ *(e)* Show that $X$ implies some intermediate statement $Z$, and then show that $Z \implies Y$.

☐ *(f)* Assume that $X$ is true, and then use this to show that $Y$ is true.

☐ *(g)* Show that either $X$ is false, or $Y$ is true, or both.

**(2)** Let $X, Y, Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Y$ implies $Z$. If we also know that $Y$ is false, we can conclude that

☐ *(a)* None of the above conclusions can be drawn.

☐ *(b)* $Z$ is false and $X$ implies $Z$. Correct Answer. $X$ is false and $X$ implies $Z$.

☐ *(c)* $X$ is false.

☐ *(d)* $Z$ is false.

☐ *(e)* $X$ is false and $Z$ is false and $X$ implies $Z$.

☐ *(f)* $X$ implies $Z$.

**(3)** Let $P(n, m)$ be a property about two integers $n$ and $m$. If we want to prove that "For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is true", then we should do the following:

☐ *(a)* Find an integer $m$ such that $P(n, m)$ is true for every integer $n$.

☐ *(b)* Find an integer $n$ such that $P(n, m)$ is true for every integer $m$.

☐ *(c)* Let $n$ be an arbitrary integer. Then find an integer $m$ (possibly depending on $n$) such that $P(n, m)$ is true.

☐ *(d)* Find an integer $n$ and an integer $m$ such that $P(n, m)$ is true.

☐ *(e)* Let $n$ and $m$ be arbitrary integers. Then show that $P(n, m)$ is true.

☐ *(f)* Show that whenever $P(n, m)$ is true, then $n$ and $m$ are integers.

☐ *(g)* Let $m$ be an arbitrary integer. Then find an integer $n$ (possibly depending on $m$) such that $P(n, m)$ is true.

**(4)** Suppose one wishes to prove that ”if all $X$ are $Y$, then all $Z$ are $W$”. To do this, it would suffice to show that

- ☐ *(a)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(b)* All $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(c)* All $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(d)* All $X$ are $Z$, and all $W$ are $Y$.
- ☐ *(e)* All $Z$ are $Y$, and all $X$ are $W$.
- ☐ *(f)* All $Y$ are $X$, and all $Z$ are $W$.
- ☐ *(g)* All $Y$ are $Z$, and all $W$ are $X$.

**(5)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that ”$P(x)$ is true for some $x$ of type $X$”, then we have to

- ☐ *(a)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type $X$.
- ☐ *(b)* Assume that $P(x)$ is true for every $x$ in $X$, and derive a contradiction.
- ☐ *(c)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.
- ☐ *(d)* Show that for every $x$ in $X$, $P(x)$ is false.
- ☐ *(e)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(f)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.
- ☐ *(g)* Show that there are no objects $x$ of type $X$.

**(6)** Let $X, Y, Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Z$ implies $X$. If we also know that $Y$ is false, we can conclude that

- ☐ *(a)* $Z$ implies $Y$.
- ☐ *(b)* $X$ is false.
- ☐ *(c)* $X$ is false and $Z$ implies $Y$.
- ☐ *(d)* $Z$ is false.
- ☐ *(e)* $X$ is false, $Z$ is false, and $Z$ implies $Y$.
- ☐ *(f)* $Z$ is false and $Z$ implies $Y$.
- ☐ *(g)* None of the above conclusions can be drawn.

**(7)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that "$P(x)$ is true for all $x$ of type $X$", then we have to

- ☐ *(a)* Show that there are no objects $x$ of type $X$.
- ☐ *(b)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.
- ☐ *(c)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type $X$.
- ☐ *(d)* Assume there exists an $x$ of type $X$ for which $P(x)$ is true, and derive a contradiction.
- ☐ *(e)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(f)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.
- ☐ *(g)* Show that for every $x$ in $X$, $P(x)$ is false.

**(8)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "$X \implies Y$", we need to show that

- ☐ *(a)* At least one of $X$ and $Y$ is false.
- ☐ *(b)* Exactly one of $X$ and $Y$ are false.
- ☐ *(c)* $X$ is true, but $Y$ is false.
- ☐ *(d)* $X$ is false.
- ☐ *(e)* $X$ and $Y$ are both false.
- ☐ *(f)* $Y$ is false.
- ☐ *(g)* $Y$ is true, but $X$ is false.

**(9)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "At least one of $X$ and $Y$ are true", we need to show that

- ☐ *(a)* $Y$ is false.
- ☐ *(b)* $X$ is true if and only if $Y$ is false.
- ☐ *(c)* Exactly one of $X$ and $Y$ are false.
- ☐ *(d)* $X$ does not imply $Y$, and $Y$ does not imply $X$.
- ☐ *(e)* $X$ is false.
- ☐ *(f)* At least one of $X$ and $Y$ are false.
- ☐ *(g)* $X$ and $Y$ are both false.

**(10)** Let $P(n,m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that ”For every integer $n$, there exists an integer $m$ such that $P(n,m)$ is true”, then we need to prove that

- ☐ *(a)* There exists an integer $m$ such that $P(n,m)$ is false for all integers $n$.
- ☐ *(b)* For every integer $n$, and every integer $m$, the property $P(n,m)$ is false.
- ☐ *(c)* For every integer $n$, there exists an integer $m$ such that $P(n,m)$ is false.
- ☐ *(d)* There exists an integer $n$ such that $P(n,m)$ is false for all integers $m$.
- ☐ *(e)* There exists integers $n$,$m$ such that $P(n,m)$ is false.
- ☐ *(f)* If $P(n,m)$ is true, then $n$ and $m$ are not integers.
- ☐ *(g)* For every integer $m$, there exists an integer $n$ such that $P(n,m)$ is false.

**(11)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that ”Both $X$ and $Y$ are true”, we need to show that

- ☐ *(a)* $X$ is true if and only if $Y$ is false.
- ☐ *(b)* $X$ does not imply $Y$, and $Y$ does not imply $X$.
- ☐ *(c)* At least one of $X$ and $Y$ are false.
- ☐ *(d)* Exactly one of $X$ and $Y$ are false.
- ☐ *(e)* $X$ and $Y$ are both false.
- ☐ *(f)* $Y$ is false.
- ☐ *(g)* $X$ is false.

**(12)** Let $X$ and $Y$ be statements. If we know that $X$ implies $Y$, then we can also conclude that

- ☐ *(a)* If $Y$ is false, then $X$ is false.
- ☐ *(b)* If $X$ is false, then $Y$ is false.
- ☐ *(c)* $X$ is true, and $Y$ is also true.
- ☐ *(d)* If $Y$ is true, then $X$ is true.
- ☐ *(e)* At least one of $X$ and $Y$ is true.
- ☐ *(f)* $X$ cannot be false.
- ☐ *(g)* $Y$ cannot be false.

**(13)** Let $P(n, m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that "There exists an integer $n$ such that $P(n, m)$ is true for all integers $m$", then we need to prove that

- ☐ *(a)* There exists an integer $m$ such that $P(n, m)$ is false for all integers $n$.
- ☐ *(b)* There exists integers $n$,$m$ such that $P(n, m)$ is false.
- ☐ *(c)* For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is false.
- ☐ *(d)* There exists an integer $n$ such that $P(n, m)$ is false for all integers $m$.
- ☐ *(e)* For every integer $n$, and every integer $m$, the property $P(n, m)$ is false.
- ☐ *(f)* If $P(n, m)$ is true, then $n$ and $m$ are not integers.
- ☐ *(g)* For every integer $m$, there exists an integer $n$ such that $P(n, m)$ is false.

**(14)** Suppose one wishes to prove that "if some $X$ are $Y$, then some $Z$ are $W$". To do this, it would suffice to show that

- ☐ *(a)* Some $Z$ are $X$, and some $Y$ are $W$.
- ☐ *(b)* Some $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(c)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(d)* All $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(e)* All $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(f)* Some $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(g)* All $X$ are $Z$, and some $Y$ are $W$.

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**Last Name**:........................**First name**:........................**Signature**:........................

**Mark the answers of the multiple-choice questions**

**(1)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

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**(5)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

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**(13)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

**(14)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

**(1)** Let $P(n, m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that "For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is true", then we need to prove that

- ☐ *(a)* There exists an integer $n$ such that $P(n, m)$ is false for all integers $m$.
- ☐ *(b)* There exists an integer $m$ such that $P(n, m)$ is false for all integers $n$.
- ☐ *(c)* There exists integers $n$,$m$ such that $P(n, m)$ is false.
- ☐ *(d)* For every integer $n$, and every integer $m$, the property $P(n, m)$ is false.
- ☐ *(e)* For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is false.
- ☐ *(f)* For every integer $m$, there exists an integer $n$ such that $P(n, m)$ is false.
- ☐ *(g)* If $P(n, m)$ is true, then $n$ and $m$ are not integers.

**(2)** Let $X$, $Y$, $Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Y$ implies $Z$. If we also know that $Y$ is false, we can conclude that

- ☐ *(a)* $Z$ is false and $X$ implies $Z$. Correct Answer. $X$ is false and $X$ implies $Z$.
- ☐ *(b)* $X$ is false.
- ☐ *(c)* None of the above conclusions can be drawn.
- ☐ *(d)* $X$ implies $Z$.
- ☐ *(e)* $Z$ is false.
- ☐ *(f)* $X$ is false and $Z$ is false and $X$ implies $Z$.

**(3)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that "$P(x)$ is true for all $x$ of type $X$", then we have to

- ☐ *(a)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type $X$.
- ☐ *(b)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.
- ☐ *(c)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(d)* Assume there exists an $x$ of type $X$ for which $P(x)$ is true, and derive a contradiction.
- ☐ *(e)* Show that for every $x$ in $X$, $P(x)$ is false.
- ☐ *(f)* Show that there are no objects $x$ of type $X$.
- ☐ *(g)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.

**(4)** Let $X$, $Y$, $Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Y$ implies $Z$. If we also know that $X$ is false, we can conclude that

- ☐ *(a)* $Z$ is false.
- ☐ *(b)* $Y$ is false, $Z$ is false and $Z$ implies $X$.
- ☐ *(c)* $Y$ is false.
- ☐ *(d)* $Z$ implies $X$.
- ☐ *(e)* $Y$ is false and $Z$ is false.
- ☐ *(f)* $Y$ is false and $Z$ implies $X$.
- ☐ *(g)* No conclusion can be drawn.

**(5)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that ”Both $X$ and $Y$ are true”, we need to show that

- ☐ *(a)* $X$ is true if and only if $Y$ is false.
- ☐ *(b)* $X$ and $Y$ are both false.
- ☐ *(c)* $X$ is false.
- ☐ *(d)* Exactly one of $X$ and $Y$ are false.
- ☐ *(e)* $X$ does not imply $Y$, and $Y$ does not imply $X$.
- ☐ *(f)* $Y$ is false.
- ☐ *(g)* At least one of $X$ and $Y$ are false.

**(6)** Suppose one wishes to prove that ”if some $X$ are $Y$, then some $Z$ are $W$”. To do this, it would suffice to show that

- ☐ *(a)* Some $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(b)* All $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(c)* Some $Z$ are $X$, and some $Y$ are $W$.
- ☐ *(d)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(e)* All $X$ are $Z$, and some $Y$ are $W$.
- ☐ *(f)* All $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(g)* Some $X$ are $Z$, and all $Y$ are $W$.

**(7)** Let $X$ and $Y$ be statements. Which of the following strategies is NOT a valid way to show that "$X \implies Y$"?

- ☐ *(a)* Show that some intermediate statement $Z \implies Y$, and then show that $X \implies Z$.
- ☐ *(b)* Assume that $X$ is true, and $Y$ is false, and deduce a contradiction.
- ☐ *(c)* Show that $X$ implies some intermediate statement $Z$, and then show that $Z \implies Y$.
- ☐ *(d)* Show that either $X$ is false, or $Y$ is true, or both.
- ☐ *(e)* Assume that $Y$ is false, and then use this to show that $X$ is false.
- ☐ *(f)* Assume that $X$ is false, and $Y$ is true, and deduce a contradiction.
- ☐ *(g)* Assume that $X$ is true, and then use this to show that $Y$ is true.

**(8)** Let $P(n, m)$ be a property about two integers $n$ and $m$. If we want to prove that "For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is true", then we should do the following:

- ☐ *(a)* Let $n$ and $m$ be arbitrary integers. Then show that $P(n, m)$ is true.
- ☐ *(b)* Let $n$ be an arbitrary integer. Then find an integer $m$ (possibly depending on $n$) such that $P(n, m)$ is true.
- ☐ *(c)* Find an integer $n$ and an integer $m$ such that $P(n, m)$ is true.
- ☐ *(d)* Show that whenever $P(n, m)$ is true, then $n$ and $m$ are integers.
- ☐ *(e)* Let $m$ be an arbitrary integer. Then find an integer $n$ (possibly depending on $m$) such that $P(n, m)$ is true.
- ☐ *(f)* Find an integer $m$ such that $P(n, m)$ is true for every integer $n$.
- ☐ *(g)* Find an integer $n$ such that $P(n, m)$ is true for every integer $m$.

**(9)** Let $P(n, m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that "There exists an integer $n$ such that $P(n, m)$ is true for all integers $m$", then we need to prove that

- ☐ *(a)* There exists integers $n$,$m$ such that $P(n, m)$ is false.
- ☐ *(b)* For every integer $n$, and every integer $m$, the property $P(n, m)$ is false.
- ☐ *(c)* There exists an integer $m$ such that $P(n, m)$ is false for all integers $n$.
- ☐ *(d)* For every integer $m$, there exists an integer $n$ such that $P(n, m)$ is false.
- ☐ *(e)* There exists an integer $n$ such that $P(n, m)$ is false for all integers $m$.
- ☐ *(f)* If $P(n, m)$ is true, then $n$ and $m$ are not integers.
- ☐ *(g)* For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is false.

**(10)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that ”At least one of $X$ and $Y$ are true”, we need to show that

- ☐ *(a)* $X$ is true if and only if $Y$ is false.
- ☐ *(b)* $Y$ is false.
- ☐ *(c)* $X$ is false.
- ☐ *(d)* Exactly one of $X$ and $Y$ are false.
- ☐ *(e)* $X$ and $Y$ are both false.
- ☐ *(f)* $X$ does not imply $Y$, and $Y$ does not imply $X$.
- ☐ *(g)* At least one of $X$ and $Y$ are false.

**(11)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that ”$X \implies Y$”, we need to show that

- ☐ *(a)* Exactly one of $X$ and $Y$ are false.
- ☐ *(b)* At least one of $X$ and $Y$ is false.
- ☐ *(c)* $X$ is true, but $Y$ is false.
- ☐ *(d)* $X$ and $Y$ are both false.
- ☐ *(e)* $Y$ is true, but $X$ is false.
- ☐ *(f)* $X$ is false.
- ☐ *(g)* $Y$ is false.

**(12)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that ”$P(x)$ is true for some $x$ of type $X$”, then we have to

- ☐ *(a)* Show that for every $x$ in $X$, $P(x)$ is false.
- ☐ *(b)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(c)* Show that there are no objects $x$ of type $X$.
- ☐ *(d)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type $X$.
- ☐ *(e)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.
- ☐ *(f)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.
- ☐ *(g)* Assume that $P(x)$ is true for every $x$ in $X$, and derive a contradiction.

**(13)** Let $X$ and $Y$ be statements. If we know that $X$ implies $Y$, then we can also conclude that

- ☐ *(a)* If $X$ is false, then $Y$ is false.
- ☐ *(b)* $Y$ cannot be false.
- ☐ *(c)* If $Y$ is false, then $X$ is false.
- ☐ *(d)* At least one of $X$ and $Y$ is true.

- ☐ *(e)* $X$ cannot be false.
- ☐ *(f)* If $Y$ is true, then $X$ is true.
- ☐ *(g)* $X$ is true, and $Y$ is also true.

**(14)** Suppose one wishes to prove that ”if all $X$ are $Y$, then all $Z$ are $W$”. To do this, it would suffice to show that

- ☐ *(a)* All $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(b)* All $Y$ are $X$, and all $Z$ are $W$.
- ☐ *(c)* All $X$ are $Z$, and all $W$ are $Y$.
- ☐ *(d)* All $Z$ are $Y$, and all $X$ are $W$.
- ☐ *(e)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(f)* All $Y$ are $Z$, and all $W$ are $X$.
- ☐ *(g)* All $X$ are $Z$, and all $Y$ are $W$.

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*Instructions: fill **completely** the bubbles with the digits of the SID (one for each column); in the lower part of the sheet, fill **completely** the bubbles with the correct answers to the corresponding question. Use a black or dark blue pen or pencil, trying to fill completely the inside of the bubble. Write only in the designated areas.*

**Last Name**:........................ **First name**:........................ **Signature**:........................

**Mark the answers of the multiple-choice questions**

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**(14)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

**(1)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that "$P(x)$ is true for some $x$ of type $X$", then we have to

- ☐ *(a)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(b)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type $X$.
- ☐ *(c)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.
- ☐ *(d)* Assume that $P(x)$ is true for every $x$ in $X$, and derive a contradiction.
- ☐ *(e)* Show that for every $x$ in $X$, $P(x)$ is false.
- ☐ *(f)* Show that there are no objects $x$ of type $X$.
- ☐ *(g)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.

**(2)** Let $X$ and $Y$ be statements. If we know that $X$ implies $Y$, then we can also conclude that

- ☐ *(a)* If $Y$ is true, then $X$ is true.
- ☐ *(b)* If $X$ is false, then $Y$ is false.
- ☐ *(c)* $Y$ cannot be false.
- ☐ *(d)* $X$ cannot be false.
- ☐ *(e)* If $Y$ is false, then $X$ is false.
- ☐ *(f)* $X$ is true, and $Y$ is also true.
- ☐ *(g)* At least one of $X$ and $Y$ is true.

**(3)** Let $X$, $Y$, $Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Y$ implies $Z$. If we also know that $Y$ is false, we can conclude that

- ☐ *(a)* $X$ is false.
- ☐ *(b)* $X$ is false and $Z$ is false and $X$ implies $Z$.
- ☐ *(c)* None of the above conclusions can be drawn.
- ☐ *(d)* $Z$ is false and $X$ implies $Z$. Correct Answer. $X$ is false and $X$ implies $Z$.
- ☐ *(e)* $X$ implies $Z$.
- ☐ *(f)* $Z$ is false.

**(4)** Let $P(n,m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that "For every integer $n$, there exists an integer $m$ such that $P(n,m)$ is true", then we need to prove that

- ☐ *(a)* For every integer $n$, there exists an integer $m$ such that $P(n,m)$ is false.
- ☐ *(b)* If $P(n,m)$ is true, then $n$ and $m$ are not integers.
- ☐ *(c)* There exists integers $n$,$m$ such that $P(n,m)$ is false.
- ☐ *(d)* For every integer $m$, there exists an integer $n$ such that $P(n,m)$ is false.
- ☐ *(e)* For every integer $n$, and every integer $m$, the property $P(n,m)$ is false.
- ☐ *(f)* There exists an integer $m$ such that $P(n,m)$ is false for all integers $n$.
- ☐ *(g)* There exists an integer $n$ such that $P(n,m)$ is false for all integers $m$.

**(5)** Suppose one wishes to prove that "if all $X$ are $Y$, then all $Z$ are $W$". To do this, it would suffice to show that

- ☐ *(a)* All $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(b)* All $Y$ are $Z$, and all $W$ are $X$.
- ☐ *(c)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(d)* All $X$ are $Z$, and all $W$ are $Y$.
- ☐ *(e)* All $Z$ are $Y$, and all $X$ are $W$.
- ☐ *(f)* All $Y$ are $X$, and all $Z$ are $W$.
- ☐ *(g)* All $X$ are $Z$, and all $Y$ are $W$.

**(6)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that "$P(x)$ is true for all $x$ of type $X$", then we have to

- ☐ *(a)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(b)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.
- ☐ *(c)* Show that there are no objects $x$ of type $X$.
- ☐ *(d)* Assume there exists an $x$ of type $X$ for which $P(x)$ is true, and derive a contradiction.
- ☐ *(e)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.
- ☐ *(f)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type $X$.
- ☐ *(g)* Show that for every $x$ in $X$, $P(x)$ is false.

**(7)** Let $X$ and $Y$ be statements. Which of the following strategies is NOT a valid way to show that "$X \implies Y$"?

- ☐ *(a)* Show that $X$ implies some intermediate statement $Z$, and then show that $Z \implies Y$.
- ☐ *(b)* Assume that $Y$ is false, and then use this to show that $X$ is false.
- ☐ *(c)* Assume that $X$ is true, and then use this to show that $Y$ is true.
- ☐ *(d)* Assume that $X$ is false, and $Y$ is true, and deduce a contradiction.
- ☐ *(e)* Show that either $X$ is false, or $Y$ is true, or both.
- ☐ *(f)* Assume that $X$ is true, and $Y$ is false, and deduce a contradiction.
- ☐ *(g)* Show that some intermediate statement $Z \implies Y$, and then show that $X \implies Z$.

**(8)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "Both $X$ and $Y$ are true", we need to show that

- ☐ *(a)* $Y$ is false.
- ☐ *(b)* $X$ is true if and only if $Y$ is false.
- ☐ *(c)* $X$ and $Y$ are both false.
- ☐ *(d)* $X$ is false.
- ☐ *(e)* $X$ does not imply $Y$, and $Y$ does not imply $X$.
- ☐ *(f)* At least one of $X$ and $Y$ are false.
- ☐ *(g)* Exactly one of $X$ and $Y$ are false.

**(9)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "At least one of $X$ and $Y$ are true", we need to show that

- ☐ *(a)* $X$ is false.
- ☐ *(b)* $X$ and $Y$ are both false.
- ☐ *(c)* $Y$ is false.
- ☐ *(d)* At least one of $X$ and $Y$ are false.
- ☐ *(e)* Exactly one of $X$ and $Y$ are false.
- ☐ *(f)* $X$ is true if and only if $Y$ is false.
- ☐ *(g)* $X$ does not imply $Y$, and $Y$ does not imply $X$.

**(10)** Let $X$, $Y$, $Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Y$ implies $Z$. If we also know that $X$ is false, we can conclude that

- ☐ *(a)* $Y$ is false and $Z$ implies $X$.
- ☐ *(b)* $Z$ is false.
- ☐ *(c)* $Y$ is false, $Z$ is false and $Z$ implies $X$.
- ☐ *(d)* $Y$ is false.
- ☐ *(e)* $Y$ is false and $Z$ is false.
- ☐ *(f)* $Z$ implies $X$.
- ☐ *(g)* No conclusion can be drawn.

**(11)** Suppose one wishes to prove that "if some $X$ are $Y$, then some $Z$ are $W$". To do this, it would suffice to show that

- ☐ *(a)* All $X$ are $Z$, and some $Y$ are $W$.
- ☐ *(b)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(c)* Some $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(d)* All $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(e)* Some $Z$ are $X$, and some $Y$ are $W$.
- ☐ *(f)* All $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(g)* Some $X$ are $Z$, and all $Y$ are $W$.

**(12)** Let $P(n,m)$ be a property about two integers $n$ and $m$. If we want to prove that "For every integer $n$, there exists an integer $m$ such that $P(n,m)$ is true", then we should do the following:

- ☐ *(a)* Show that whenever $P(n,m)$ is true, then $n$ and $m$ are integers.
- ☐ *(b)* Let $n$ be an arbitrary integer. Then find an integer $m$ (possibly depending on $n$) such that $P(n,m)$ is true.
- ☐ *(c)* Let $n$ and $m$ be arbitrary integers. Then show that $P(n,m)$ is true.
- ☐ *(d)* Let $m$ be an arbitrary integer. Then find an integer $n$ (possibly depending on $m$) such that $P(n,m)$ is true.
- ☐ *(e)* Find an integer $n$ such that $P(n,m)$ is true for every integer $m$.
- ☐ *(f)* Find an integer $n$ and an integer $m$ such that $P(n,m)$ is true.
- ☐ *(g)* Find an integer $m$ such that $P(n,m)$ is true for every integer $n$.

**(13)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "$X \implies Y$", we need to show that

- ☐ *(a)* $Y$ is false.
- ☐ *(b)* $X$ and $Y$ are both false.
- ☐ *(c)* $X$ is true, but $Y$ is false.
- ☐ *(d)* $Y$ is true, but $X$ is false.
- ☐ *(e)* $X$ is false.
- ☐ *(f)* Exactly one of $X$ and $Y$ are false.
- ☐ *(g)* At least one of $X$ and $Y$ is false.

**(14)** Let $P(n,m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that "There exists an integer $n$ such that $P(n,m)$ is true for all integers $m$", then we need to prove that

- ☐ *(a)* There exists an integer $n$ such that $P(n,m)$ is false for all integers $m$.
- ☐ *(b)* For every integer $n$, there exists an integer $m$ such that $P(n,m)$ is false.
- ☐ *(c)* For every integer $m$, there exists an integer $n$ such that $P(n,m)$ is false.
- ☐ *(d)* There exists integers $n$,$m$ such that $P(n,m)$ is false.
- ☐ *(e)* For every integer $n$, and every integer $m$, the property $P(n,m)$ is false.
- ☐ *(f)* There exists an integer $m$ such that $P(n,m)$ is false for all integers $n$.
- ☐ *(g)* If $P(n,m)$ is true, then $n$ and $m$ are not integers.

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**Last Name**:......................... **First name**:......................... **Signature**:..........................

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**(1)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that "$P(x)$ is true for some $x$ of type $X$", then we have to

- ☐ *(a)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(b)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.
- ☐ *(c)* Assume that $P(x)$ is true for every $x$ in $X$, and derive a contradiction.
- ☐ *(d)* Show that for every $x$ in $X$, $P(x)$ is false.
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- ☐ *(f)* Show that there are no objects $x$ of type $X$.
- ☐ *(g)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.

**(2)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "$X \implies Y$", we need to show that

- ☐ *(a)* $X$ is true, but $Y$ is false.
- ☐ *(b)* Exactly one of $X$ and $Y$ are false.
- ☐ *(c)* $X$ and $Y$ are both false.
- ☐ *(d)* $Y$ is true, but $X$ is false.
- ☐ *(e)* At least one of $X$ and $Y$ is false.
- ☐ *(f)* $X$ is false.
- ☐ *(g)* $Y$ is false.

**(3)** Suppose one wishes to prove that "if all $X$ are $Y$, then all $Z$ are $W$". To do this, it would suffice to show that

- ☐ *(a)* All $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(b)* All $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(c)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(d)* All $X$ are $Z$, and all $W$ are $Y$.
- ☐ *(e)* All $Y$ are $X$, and all $Z$ are $W$.
- ☐ *(f)* All $Z$ are $Y$, and all $X$ are $W$.
- ☐ *(g)* All $Y$ are $Z$, and all $W$ are $X$.

**(4)** Let $P(n,m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that "For every integer $n$, there exists an integer $m$ such that $P(n,m)$ is true", then we need to prove that

- ☐ *(a)* There exists an integer $n$ such that $P(n,m)$ is false for all integers $m$.
- ☐ *(b)* For every integer $m$, there exists an integer $n$ such that $P(n,m)$ is false.
- ☐ *(c)* There exists an integer $m$ such that $P(n,m)$ is false for all integers $n$.
- ☐ *(d)* There exists integers $n$,$m$ such that $P(n,m)$ is false.
- ☐ *(e)* For every integer $n$, and every integer $m$, the property $P(n,m)$ is false.
- ☐ *(f)* If $P(n,m)$ is true, then $n$ and $m$ are not integers.
- ☐ *(g)* For every integer $n$, there exists an integer $m$ such that $P(n,m)$ is false.

**(5)** Let $P(n,m)$ be a property about two integers $n$ and $m$. If we want to prove that "For every integer $n$, there exists an integer $m$ such that $P(n,m)$ is true", then we should do the following:

- ☐ *(a)* Find an integer $n$ such that $P(n,m)$ is true for every integer $m$.
- ☐ *(b)* Find an integer $m$ such that $P(n,m)$ is true for every integer $n$.
- ☐ *(c)* Let $n$ and $m$ be arbitrary integers. Then show that $P(n,m)$ is true.
- ☐ *(d)* Let $m$ be an arbitrary integer. Then find an integer $n$ (possibly depending on $m$) such that $P(n,m)$ is true.
- ☐ *(e)* Show that whenever $P(n,m)$ is true, then $n$ and $m$ are integers.
- ☐ *(f)* Find an integer $n$ and an integer $m$ such that $P(n,m)$ is true.
- ☐ *(g)* Let $n$ be an arbitrary integer. Then find an integer $m$ (possibly depending on $n$) such that $P(n,m)$ is true.

**(6)** Let $X$ and $Y$ be statements. If we know that $X$ implies $Y$, then we can also conclude that

- ☐ *(a)* $X$ cannot be false.
- ☐ *(b)* If $X$ is false, then $Y$ is false.
- ☐ *(c)* At least one of $X$ and $Y$ is true.
- ☐ *(d)* If $Y$ is false, then $X$ is false.
- ☐ *(e)* $X$ is true, and $Y$ is also true.
- ☐ *(f)* If $Y$ is true, then $X$ is true.
- ☐ *(g)* $Y$ cannot be false.

**(7)** Suppose one wishes to prove that ”if some $X$ are $Y$, then some $Z$ are $W$”. To do this, it would suffice to show that

- ☐ *(a)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(b)* All $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(c)* All $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(d)* Some $Z$ are $X$, and some $Y$ are $W$.
- ☐ *(e)* Some $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(f)* All $X$ are $Z$, and some $Y$ are $W$.
- ☐ *(g)* Some $Z$ are $X$, and all $Y$ are $W$.

**(8)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that ”Both $X$ and $Y$ are true”, we need to show that

- ☐ *(a)* $X$ and $Y$ are both false.
- ☐ *(b)* At least one of $X$ and $Y$ are false.
- ☐ *(c)* $X$ is false.
- ☐ *(d)* Exactly one of $X$ and $Y$ are false.
- ☐ *(e)* $X$ does not imply $Y$, and $Y$ does not imply $X$.
- ☐ *(f)* $Y$ is false.
- ☐ *(g)* $X$ is true if and only if $Y$ is false.

**(9)** Let $X$, $Y$, $Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Z$ implies $X$. If we also know that $Y$ is false, we can conclude that

- ☐ *(a)* $Z$ implies $Y$.
- ☐ *(b)* $X$ is false.
- ☐ *(c)* $X$ is false, $Z$ is false, and $Z$ implies $Y$.
- ☐ *(d)* $Z$ is false and $Z$ implies $Y$.
- ☐ *(e)* $X$ is false and $Z$ implies $Y$.
- ☐ *(f)* $Z$ is false.
- ☐ *(g)* None of the above conclusions can be drawn.

**(10)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "At least one of $X$ and $Y$ are true", we need to show that

- ☐ *(a)* $X$ and $Y$ are both false.
- ☐ *(b)* At least one of $X$ and $Y$ are false.
- ☐ *(c)* $Y$ is false.
- ☐ *(d)* $X$ does not imply $Y$, and $Y$ does not imply $X$.
- ☐ *(e)* $X$ is true if and only if $Y$ is false.
- ☐ *(f)* $X$ is false.
- ☐ *(g)* Exactly one of $X$ and $Y$ are false.

**(11)** Let $X, Y, Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Y$ implies $Z$. If we also know that $Y$ is false, we can conclude that

- ☐ *(a)* $X$ implies $Z$.
- ☐ *(b)* $X$ is false and $Z$ is false and $X$ implies $Z$.
- ☐ *(c)* $Z$ is false.
- ☐ *(d)* $Z$ is false and $X$ implies $Z$. Correct Answer. $X$ is false and $X$ implies $Z$.
- ☐ *(e)* $X$ is false.
- ☐ *(f)* None of the above conclusions can be drawn.

**(12)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that "$P(x)$ is true for all $x$ of type $X$", then we have to

- ☐ *(a)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type $X$.
- ☐ *(b)* Assume there exists an $x$ of type $X$ for which $P(x)$ is true, and derive a contradiction.
- ☐ *(c)* Show that for every $x$ in $X$, $P(x)$ is false.
- ☐ *(d)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(e)* Show that there are no objects $x$ of type $X$.
- ☐ *(f)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.
- ☐ *(g)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.

**(13)** Let $P(n, m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that "There exists an integer $n$ such that $P(n, m)$ is true for all integers $m$", then we need to prove that

- ☐ *(a)* If $P(n, m)$ is true, then $n$ and $m$ are not integers.
- ☐ *(b)* For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is false.
- ☐ *(c)* There exists integers $n$,$m$ such that $P(n, m)$ is false.
- ☐ *(d)* There exists an integer $n$ such that $P(n, m)$ is false for all integers $m$.
- ☐ *(e)* There exists an integer $m$ such that $P(n, m)$ is false for all integers $n$.
- ☐ *(f)* For every integer $n$, and every integer $m$, the property $P(n, m)$ is false.
- ☐ *(g)* For every integer $m$, there exists an integer $n$ such that $P(n, m)$ is false.

**(14)** Let $X$ and $Y$ be statements. Which of the following strategies is NOT a valid way to show that "$X \implies Y$"?

- ☐ *(a)* Show that either $X$ is false, or $Y$ is true, or both.
- ☐ *(b)* Assume that $X$ is true, and $Y$ is false, and deduce a contradiction.
- ☐ *(c)* Show that $X$ implies some intermediate statement $Z$, and then show that $Z \implies Y$.
- ☐ *(d)* Show that some intermediate statement $Z \implies Y$, and then show that $X \implies Z$.
- ☐ *(e)* Assume that $X$ is true, and then use this to show that $Y$ is true.
- ☐ *(f)* Assume that $Y$ is false, and then use this to show that $X$ is false.
- ☐ *(g)* Assume that $X$ is false, and $Y$ is true, and deduce a contradiction.

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| ⑥ | ⑥ | ⑥ | ⑥ | ⑥ | ⑥ |
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*Instructions: fill* ***completely*** *the bubbles with the digits of the SID (one for each column); in the lower part of the sheet, fill* ***completely*** *the bubbles with the correct answers to the corresponding question. Use a black or dark blue pen or pencil, trying to fill completely the inside of the bubble. Write only in the designated areas.*

**Last Name**:. . . . . . . . . . . . . . . . . . . . . . . . **First name**:. . . . . . . . . . . . . . . . . . . . . . . . **Signature**:. . . . . . . . . . . . . . . . . . . . . . . . .

**Mark the answers of the multiple-choice questions**

**(1)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

**(2)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

**(3)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

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**(10)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

**(11)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

**(12)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

**(13)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

**(14)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

**(1)** Let $P(n, m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that "For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is true", then we need to prove that

- ☐ *(a)* There exists integers $n$,$m$ such that $P(n, m)$ is false.
- ☐ *(b)* There exists an integer $n$ such that $P(n, m)$ is false for all integers $m$.
- ☐ *(c)* For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is false.
- ☐ *(d)* There exists an integer $m$ such that $P(n, m)$ is false for all integers $n$.
- ☐ *(e)* If $P(n, m)$ is true, then $n$ and $m$ are not integers.
- ☐ *(f)* For every integer $n$, and every integer $m$, the property $P(n, m)$ is false.
- ☐ *(g)* For every integer $m$, there exists an integer $n$ such that $P(n, m)$ is false.

**(2)** Let $X$, $Y$, $Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Y$ implies $Z$. If we also know that $X$ is false, we can conclude that

- ☐ *(a)* $Y$ is false and $Z$ is false.
- ☐ *(b)* No conclusion can be drawn.
- ☐ *(c)* $Y$ is false and $Z$ implies $X$.
- ☐ *(d)* $Y$ is false, $Z$ is false and $Z$ implies $X$.
- ☐ *(e)* $Z$ implies $X$.
- ☐ *(f)* $Y$ is false.
- ☐ *(g)* $Z$ is false.

**(3)** Let $X$ and $Y$ be statements. If we know that $X$ implies $Y$, then we can also conclude that

- ☐ *(a)* $Y$ cannot be false.
- ☐ *(b)* If $Y$ is true, then $X$ is true.
- ☐ *(c)* $X$ cannot be false.
- ☐ *(d)* $X$ is true, and $Y$ is also true.
- ☐ *(e)* At least one of $X$ and $Y$ is true.
- ☐ *(f)* If $Y$ is false, then $X$ is false.
- ☐ *(g)* If $X$ is false, then $Y$ is false.

**(4)** Let $X$ and $Y$ be statements. Which of the following strategies is NOT a valid way to show that "$X \implies Y$"?

- ☐ *(a)* Show that either $X$ is false, or $Y$ is true, or both.
- ☐ *(b)* Assume that $Y$ is false, and then use this to show that $X$ is false.
- ☐ *(c)* Assume that $X$ is false, and $Y$ is true, and deduce a contradiction.
- ☐ *(d)* Assume that $X$ is true, and then use this to show that $Y$ is true.
- ☐ *(e)* Show that $X$ implies some intermediate statement $Z$, and then show that $Z \implies Y$.
- ☐ *(f)* Assume that $X$ is true, and $Y$ is false, and deduce a contradiction.
- ☐ *(g)* Show that some intermediate statement $Z \implies Y$, and then show that $X \implies Z$.

**(5)** Let $X$, $Y$, $Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Y$ implies $Z$. If we also know that $Y$ is false, we can conclude that

- ☐ *(a)* $Z$ is false.
- ☐ *(b)* $X$ is false and $Z$ is false and $X$ implies $Z$.
- ☐ *(c)* $X$ implies $Z$.
- ☐ *(d)* $X$ is false.
- ☐ *(e)* $Z$ is false and $X$ implies $Z$. Correct Answer. $X$ is false and $X$ implies $Z$.
- ☐ *(f)* None of the above conclusions can be drawn.

**(6)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "Both $X$ and $Y$ are true", we need to show that

- ☐ *(a)* At least one of $X$ and $Y$ are false.
- ☐ *(b)* $X$ does not imply $Y$, and $Y$ does not imply $X$.
- ☐ *(c)* $Y$ is false.
- ☐ *(d)* $X$ is true if and only if $Y$ is false.
- ☐ *(e)* $X$ is false.
- ☐ *(f)* Exactly one of $X$ and $Y$ are false.
- ☐ *(g)* $X$ and $Y$ are both false.

**(7)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "$X \implies Y$", we need to show that

- ☐ *(a)* Exactly one of $X$ and $Y$ are false.
- ☐ *(b)* At least one of $X$ and $Y$ is false.
- ☐ *(c)* $X$ is true, but $Y$ is false.
- ☐ *(d)* $X$ and $Y$ are both false.
- ☐ *(e)* $Y$ is true, but $X$ is false.
- ☐ *(f)* $X$ is false.
- ☐ *(g)* $Y$ is false.

**(8)** Let $P(n, m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that "There exists an integer $n$ such that $P(n, m)$ is true for all integers $m$", then we need to prove that

- ☐ *(a)* For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is false.
- ☐ *(b)* There exists an integer $m$ such that $P(n, m)$ is false for all integers $n$.
- ☐ *(c)* If $P(n, m)$ is true, then $n$ and $m$ are not integers.
- ☐ *(d)* There exists an integer $n$ such that $P(n, m)$ is false for all integers $m$.
- ☐ *(e)* For every integer $m$, there exists an integer $n$ such that $P(n, m)$ is false.
- ☐ *(f)* There exists integers $n$,$m$ such that $P(n, m)$ is false.
- ☐ *(g)* For every integer $n$, and every integer $m$, the property $P(n, m)$ is false.

**(9)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that "$P(x)$ is true for some $x$ of type $X$", then we have to

- ☐ *(a)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type $X$.
- ☐ *(b)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(c)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.
- ☐ *(d)* Assume that $P(x)$ is true for every $x$ in $X$, and derive a contradiction.
- ☐ *(e)* Show that for every $x$ in $X$, $P(x)$ is false.
- ☐ *(f)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.
- ☐ *(g)* Show that there are no objects $x$ of type $X$.

**(10)** Suppose one wishes to prove that "if some $X$ are $Y$, then some $Z$ are $W$". To do this, it would suffice to show that

- ☐ *(a)* Some $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(b)* All $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(c)* Some $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(d)* All $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(e)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(f)* Some $Z$ are $X$, and some $Y$ are $W$.
- ☐ *(g)* All $X$ are $Z$, and some $Y$ are $W$.

**(11)** Suppose one wishes to prove that "if all $X$ are $Y$, then all $Z$ are $W$". To do this, it would suffice to show that

- ☐ *(a)* All $Y$ are $X$, and all $Z$ are $W$.
- ☐ *(b)* All $Y$ are $Z$, and all $W$ are $X$.
- ☐ *(c)* All $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(d)* All $Z$ are $Y$, and all $X$ are $W$.
- ☐ *(e)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(f)* All $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(g)* All $X$ are $Z$, and all $W$ are $Y$.

**(12)** Let $P(n, m)$ be a property about two integers $n$ and $m$. If we want to prove that "For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is true", then we should do the following:

- ☐ *(a)* Find an integer $n$ such that $P(n, m)$ is true for every integer $m$.
- ☐ *(b)* Let $n$ be an arbitrary integer. Then find an integer $m$ (possibly depending on $n$) such that $P(n, m)$ is true.
- ☐ *(c)* Find an integer $m$ such that $P(n, m)$ is true for every integer $n$.
- ☐ *(d)* Let $m$ be an arbitrary integer. Then find an integer $n$ (possibly depending on $m$) such that $P(n, m)$ is true.
- ☐ *(e)* Let $n$ and $m$ be arbitrary integers. Then show that $P(n, m)$ is true.
- ☐ *(f)* Show that whenever $P(n, m)$ is true, then $n$ and $m$ are integers.
- ☐ *(g)* Find an integer $n$ and an integer $m$ such that $P(n, m)$ is true.

**(13)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that "$P(x)$ is true for all $x$ of type $X$", then we have to

- ☐ *(a)* Show that for every $x$ in $X$, $P(x)$ is false.
- ☐ *(b)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.
- ☐ *(c)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.
- ☐ *(d)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type $X$.
- ☐ *(e)* Show that there are no objects $x$ of type $X$.
- ☐ *(f)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(g)* Assume there exists an $x$ of type $X$ for which $P(x)$ is true, and derive a contradiction.

**(14)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "At least one of $X$ and $Y$ are true", we need to show that

- ☐ *(a)* $X$ does not imply $Y$, and $Y$ does not imply $X$.
- ☐ *(b)* $X$ is true if and only if $Y$ is false.
- ☐ *(c)* $Y$ is false.
- ☐ *(d)* Exactly one of $X$ and $Y$ are false.
- ☐ *(e)* $X$ is false.
- ☐ *(f)* At least one of $X$ and $Y$ are false.
- ☐ *(g)* $X$ and $Y$ are both false.

**Student ID**:

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**Last Name**:........................ **First name**:........................ **Signature**:........................

**Mark the answers of the multiple-choice questions**

**(1)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

**(2)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

**(3)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

**(4)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

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**(14)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

**(1)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that "$P(x)$ is true for some $x$ of type $X$", then we have to

- ☐ *(a)* Assume that $P(x)$ is true for every $x$ in $X$, and derive a contradiction.
- ☐ *(b)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type $X$.
- ☐ *(c)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.
- ☐ *(d)* Show that for every $x$ in $X$, $P(x)$ is false.
- ☐ *(e)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(f)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.
- ☐ *(g)* Show that there are no objects $x$ of type $X$.

**(2)** Let $P(n, m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that "For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is true", then we need to prove that

- ☐ *(a)* There exists integers $n$,$m$ such that $P(n, m)$ is false.
- ☐ *(b)* For every integer $m$, there exists an integer $n$ such that $P(n, m)$ is false.
- ☐ *(c)* For every integer $n$, and every integer $m$, the property $P(n, m)$ is false.
- ☐ *(d)* For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is false.
- ☐ *(e)* There exists an integer $m$ such that $P(n, m)$ is false for all integers $n$.
- ☐ *(f)* There exists an integer $n$ such that $P(n, m)$ is false for all integers $m$.
- ☐ *(g)* If $P(n, m)$ is true, then $n$ and $m$ are not integers.

**(3)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "Both $X$ and $Y$ are true", we need to show that

- ☐ *(a)* Exactly one of $X$ and $Y$ are false.
- ☐ *(b)* $Y$ is false.
- ☐ *(c)* At least one of $X$ and $Y$ are false.
- ☐ *(d)* $X$ does not imply $Y$, and $Y$ does not imply $X$.
- ☐ *(e)* $X$ is false.
- ☐ *(f)* $X$ is true if and only if $Y$ is false.
- ☐ *(g)* $X$ and $Y$ are both false.

**(4)** Suppose one wishes to prove that "if all $X$ are $Y$, then all $Z$ are $W$". To do this, it would suffice to show that

- ☐ *(a)* All $Z$ are $Y$, and all $X$ are $W$.
- ☐ *(b)* All $Y$ are $Z$, and all $W$ are $X$.
- ☐ *(c)* All $Y$ are $X$, and all $Z$ are $W$.
- ☐ *(d)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(e)* All $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(f)* All $X$ are $Z$, and all $W$ are $Y$.
- ☐ *(g)* All $Z$ are $X$, and all $Y$ are $W$.

**(5)** Let $X$ and $Y$ be statements. Which of the following strategies is NOT a valid way to show that "$X \implies Y$"?

- ☐ *(a)* Show that some intermediate statement $Z \implies Y$, and then show that $X \implies Z$.
- ☐ *(b)* Assume that $Y$ is false, and then use this to show that $X$ is false.
- ☐ *(c)* Assume that $X$ is true, and then use this to show that $Y$ is true.
- ☐ *(d)* Assume that $X$ is true, and $Y$ is false, and deduce a contradiction.
- ☐ *(e)* Assume that $X$ is false, and $Y$ is true, and deduce a contradiction.
- ☐ *(f)* Show that either $X$ is false, or $Y$ is true, or both.
- ☐ *(g)* Show that $X$ implies some intermediate statement $Z$, and then show that $Z \implies Y$.

**(6)** Let $X$ and $Y$ be statements. If we know that $X$ implies $Y$, then we can also conclude that

- ☐ *(a)* If $Y$ is false, then $X$ is false.
- ☐ *(b)* $X$ is true, and $Y$ is also true.
- ☐ *(c)* If $Y$ is true, then $X$ is true.
- ☐ *(d)* $Y$ cannot be false.
- ☐ *(e)* $X$ cannot be false.
- ☐ *(f)* If $X$ is false, then $Y$ is false.
- ☐ *(g)* At least one of $X$ and $Y$ is true.

**(7)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "At least one of $X$ and $Y$ are true", we need to show that

- ☐ *(a)* $X$ does not imply $Y$, and $Y$ does not imply $X$.
- ☐ *(b)* $X$ is true if and only if $Y$ is false.
- ☐ *(c)* $Y$ is false.
- ☐ *(d)* At least one of $X$ and $Y$ are false.
- ☐ *(e)* $X$ and $Y$ are both false.
- ☐ *(f)* $X$ is false.
- ☐ *(g)* Exactly one of $X$ and $Y$ are false.

**(8)** Suppose one wishes to prove that "if some $X$ are $Y$, then some $Z$ are $W$". To do this, it would suffice to show that

- ☐ *(a)* Some $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(b)* Some $Z$ are $X$, and some $Y$ are $W$.
- ☐ *(c)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(d)* All $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(e)* All $X$ are $Z$, and some $Y$ are $W$.
- ☐ *(f)* Some $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(g)* All $Z$ are $X$, and all $Y$ are $W$.

**(9)** Let $P(n,m)$ be a property about two integers $n$ and $m$. If we want to prove that "For every integer $n$, there exists an integer $m$ such that $P(n,m)$ is true", then we should do the following:

- ☐ *(a)* Let $n$ and $m$ be arbitrary integers. Then show that $P(n,m)$ is true.
- ☐ *(b)* Let $n$ be an arbitrary integer. Then find an integer $m$ (possibly depending on $n$) such that $P(n,m)$ is true.
- ☐ *(c)* Find an integer $m$ such that $P(n,m)$ is true for every integer $n$.
- ☐ *(d)* Find an integer $n$ and an integer $m$ such that $P(n,m)$ is true.
- ☐ *(e)* Let $m$ be an arbitrary integer. Then find an integer $n$ (possibly depending on $m$) such that $P(n,m)$ is true.
- ☐ *(f)* Show that whenever $P(n,m)$ is true, then $n$ and $m$ are integers.
- ☐ *(g)* Find an integer $n$ such that $P(n,m)$ is true for every integer $m$.

**(10)** Let $X$, $Y$, $Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Y$ implies $Z$. If we also know that $X$ is false, we can conclude that

- ☐ *(a)* $Z$ implies $X$.
- ☐ *(b)* $Y$ is false, $Z$ is false and $Z$ implies $X$.
- ☐ *(c)* $Y$ is false and $Z$ implies $X$.
- ☐ *(d)* No conclusion can be drawn.
- ☐ *(e)* $Z$ is false.
- ☐ *(f)* $Y$ is false and $Z$ is false.
- ☐ *(g)* $Y$ is false.

**(11)** Let $P(x)$ be a property about some object $x$ of type X. If we want to DISPROVE the claim that "$P(x)$ is true for all $x$ of type $X$", then we have to

- ☐ *(a)* Assume there exists an $x$ of type $X$ for which $P(x)$ is true, and derive a contradiction.
- ☐ *(b)* Show that there are no objects $x$ of type $X$.
- ☐ *(c)* Show that there exists an $x$ which is not of type X, but for which $P(x)$ is still true.
- ☐ *(d)* Show that for every $x$ in X, $P(x)$ is false.
- ☐ *(e)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.
- ☐ *(f)* Show that for every $x$ in X, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(g)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type X.

**(12)** Let $P(n, m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that "There exists an integer $n$ such that $P(n, m)$ is true for all integers $m$", then we need to prove that

- ☐ *(a)* For every integer $n$, and every integer $m$, the property $P(n, m)$ is false.
- ☐ *(b)* If $P(n, m)$ is true, then $n$ and $m$ are not integers.
- ☐ *(c)* For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is false.
- ☐ *(d)* There exists an integer $n$ such that $P(n, m)$ is false for all integers $m$.
- ☐ *(e)* There exists an integer $m$ such that $P(n, m)$ is false for all integers $n$.
- ☐ *(f)* For every integer $m$, there exists an integer $n$ such that $P(n, m)$ is false.
- ☐ *(g)* There exists integers $n$,$m$ such that $P(n, m)$ is false.

**(13)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "$X \implies Y$", we need to show that

- ☐ *(a)* $X$ is false.
- ☐ *(b)* $Y$ is false.
- ☐ *(c)* $X$ is true, but $Y$ is false.
- ☐ *(d)* $X$ and $Y$ are both false.
- ☐ *(e)* At least one of $X$ and $Y$ is false.
- ☐ *(f)* $Y$ is true, but $X$ is false.
- ☐ *(g)* Exactly one of $X$ and $Y$ are false.

**(14)** Let $X$, $Y$, $Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Y$ implies $Z$. If we also know that $Y$ is false, we can conclude that

- ☐ *(a)* $Z$ is false.
- ☐ *(b)* $X$ is false and $Z$ is false and $X$ implies $Z$.
- ☐ *(c)* $Z$ is false and $X$ implies $Z$. Correct Answer. $X$ is false and $X$ implies $Z$.
- ☐ *(d)* None of the above conclusions can be drawn.
- ☐ *(e)* $X$ is false.
- ☐ *(f)* $X$ implies $Z$.

**Student ID**:

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*Instructions: fill **completely** the bubbles with the digits of the SID (one for each column); in the lower part of the sheet, fill **completely** the bubbles with the correct answers to the corresponding question. Use a black or dark blue pen or pencil, trying to fill completely the inside of the bubble. Write only in the designated areas.*

**Last Name**:. . . . . . . . . . . . . . . . . . . . . . . . **First name**:. . . . . . . . . . . . . . . . . . . . . . . . **Signature**:. . . . . . . . . . . . . . . . . . . . . . . . .

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**Mark the answers of the multiple-choice questions**

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**(14)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ

**(1)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "At least one of $X$ and $Y$ are true", we need to show that

- ☐ *(a)* $X$ does not imply $Y$, and $Y$ does not imply $X$.
- ☐ *(b)* At least one of $X$ and $Y$ are false.
- ☐ *(c)* Exactly one of $X$ and $Y$ are false.
- ☐ *(d)* $X$ is false.
- ☐ *(e)* $Y$ is false.
- ☐ *(f)* $X$ and $Y$ are both false.
- ☐ *(g)* $X$ is true if and only if $Y$ is false.

**(2)** Let $P(n, m)$ be a property about two integers $n$ and $m$. If we want to prove that "For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is true", then we should do the following:

- ☐ *(a)* Let $n$ be an arbitrary integer. Then find an integer $m$ (possibly depending on $n$) such that $P(n, m)$ is true.
- ☐ *(b)* Let $n$ and $m$ be arbitrary integers. Then show that $P(n, m)$ is true.
- ☐ *(c)* Find an integer $n$ such that $P(n, m)$ is true for every integer $m$.
- ☐ *(d)* Show that whenever $P(n, m)$ is true, then $n$ and $m$ are integers.
- ☐ *(e)* Let $m$ be an arbitrary integer. Then find an integer $n$ (possibly depending on $m$) such that $P(n, m)$ is true.
- ☐ *(f)* Find an integer $m$ such that $P(n, m)$ is true for every integer $n$.
- ☐ *(g)* Find an integer $n$ and an integer $m$ such that $P(n, m)$ is true.

**(3)** Let $X$ and $Y$ be statements. If we know that $X$ implies $Y$, then we can also conclude that

- ☐ *(a)* If $Y$ is true, then $X$ is true.
- ☐ *(b)* $X$ is true, and $Y$ is also true.
- ☐ *(c)* If $Y$ is false, then $X$ is false.
- ☐ *(d)* At least one of $X$ and $Y$ is true.
- ☐ *(e)* $X$ cannot be false.
- ☐ *(f)* $Y$ cannot be false.
- ☐ *(g)* If $X$ is false, then $Y$ is false.

**(4)** Let $P(n, m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that "For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is true", then we need to prove that

- ☐ *(a)* There exists an integer $m$ such that $P(n, m)$ is false for all integers $n$.
- ☐ *(b)* For every integer $n$, and every integer $m$, the property $P(n, m)$ is false.
- ☐ *(c)* For every integer $m$, there exists an integer $n$ such that $P(n, m)$ is false.
- ☐ *(d)* For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is false.
- ☐ *(e)* There exists an integer $n$ such that $P(n, m)$ is false for all integers $m$.
- ☐ *(f)* If $P(n, m)$ is true, then $n$ and $m$ are not integers.
- ☐ *(g)* There exists integers $n$,$m$ such that $P(n, m)$ is false.

**(5)** Let $P(n, m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that "There exists an integer $n$ such that $P(n, m)$ is true for all integers $m$", then we need to prove that

- ☐ *(a)* For every integer $n$, and every integer $m$, the property $P(n, m)$ is false.
- ☐ *(b)* For every integer $m$, there exists an integer $n$ such that $P(n, m)$ is false.
- ☐ *(c)* If $P(n, m)$ is true, then $n$ and $m$ are not integers.
- ☐ *(d)* There exists an integer $m$ such that $P(n, m)$ is false for all integers $n$.
- ☐ *(e)* There exists integers $n$,$m$ such that $P(n, m)$ is false.
- ☐ *(f)* There exists an integer $n$ such that $P(n, m)$ is false for all integers $m$.
- ☐ *(g)* For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is false.

**(6)** Let $X$ and $Y$ be statements. Which of the following strategies is NOT a valid way to show that "$X \implies Y$"?

- ☐ *(a)* Show that $X$ implies some intermediate statement $Z$, and then show that $Z \implies Y$.
- ☐ *(b)* Show that either $X$ is false, or $Y$ is true, or both.
- ☐ *(c)* Assume that $X$ is true, and $Y$ is false, and deduce a contradiction.
- ☐ *(d)* Show that some intermediate statement $Z \implies Y$, and then show that $X \implies Z$.
- ☐ *(e)* Assume that $X$ is true, and then use this to show that $Y$ is true.
- ☐ *(f)* Assume that $X$ is false, and $Y$ is true, and deduce a contradiction.
- ☐ *(g)* Assume that $Y$ is false, and then use this to show that $X$ is false.

**(7)** Suppose one wishes to prove that "if all $X$ are $Y$, then all $Z$ are $W$". To do this, it would suffice to show that

- ☐ *(a)* All $Z$ are $Y$, and all $X$ are $W$.
- ☐ *(b)* All $X$ are $Z$, and all $W$ are $Y$.
- ☐ *(c)* All $Y$ are $Z$, and all $W$ are $X$.
- ☐ *(d)* All $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(e)* All $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(f)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(g)* All $Y$ are $X$, and all $Z$ are $W$.

**(8)** Let $X$, $Y$, $Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Y$ implies $Z$. If we also know that $Y$ is false, we can conclude that

- ☐ *(a)* $X$ implies $Z$.
- ☐ *(b)* $X$ is false and $Z$ is false and $X$ implies $Z$.
- ☐ *(c)* $Z$ is false and $X$ implies $Z$. Correct Answer. $X$ is false and $X$ implies $Z$.
- ☐ *(d)* $Z$ is false.
- ☐ *(e)* None of the above conclusions can be drawn.
- ☐ *(f)* $X$ is false.

**(9)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that "$P(x)$ is true for some $x$ of type $X$", then we have to

- ☐ *(a)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.
- ☐ *(b)* Assume that $P(x)$ is true for every $x$ in $X$, and derive a contradiction.
- ☐ *(c)* Show that there are no objects $x$ of type $X$.
- ☐ *(d)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(e)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.
- ☐ *(f)* Show that for every $x$ in $X$, $P(x)$ is false.
- ☐ *(g)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type $X$.

**(10)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "$X \implies Y$", we need to show that

- ☐ *(a)* Exactly one of $X$ and $Y$ are false.
- ☐ *(b)* $X$ is false.
- ☐ *(c)* $X$ is true, but $Y$ is false.
- ☐ *(d)* At least one of $X$ and $Y$ is false.
- ☐ *(e)* $Y$ is true, but $X$ is false.
- ☐ *(f)* $X$ and $Y$ are both false.
- ☐ *(g)* $Y$ is false.

**(11)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that "$P(x)$ is true for all $x$ of type $X$", then we have to

- ☐ *(a)* Show that for every $x$ in $X$, $P(x)$ is false.
- ☐ *(b)* Assume there exists an $x$ of type $X$ for which $P(x)$ is true, and derive a contradiction.
- ☐ *(c)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.
- ☐ *(d)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.
- ☐ *(e)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type $X$.
- ☐ *(f)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(g)* Show that there are no objects $x$ of type $X$.

**(12)** Let $X$, $Y$, $Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Z$ implies $X$. If we also know that $Y$ is false, we can conclude that

- ☐ *(a)* None of the above conclusions can be drawn.
- ☐ *(b)* $X$ is false and $Z$ implies $Y$.
- ☐ *(c)* $Z$ is false and $Z$ implies $Y$.
- ☐ *(d)* $Z$ implies $Y$.
- ☐ *(e)* $X$ is false.
- ☐ *(f)* $X$ is false, $Z$ is false, and $Z$ implies $Y$.
- ☐ *(g)* $Z$ is false.

**(13)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "Both $X$ and $Y$ are true", we need to show that

- ☐ *(a)* $Y$ is false.
- ☐ *(b)* At least one of $X$ and $Y$ are false.
- ☐ *(c)* Exactly one of $X$ and $Y$ are false.
- ☐ *(d)* $X$ and $Y$ are both false.
- ☐ *(e)* $X$ is false.
- ☐ *(f)* $X$ does not imply $Y$, and $Y$ does not imply $X$.
- ☐ *(g)* $X$ is true if and only if $Y$ is false.

**(14)** Suppose one wishes to prove that "if some $X$ are $Y$, then some $Z$ are $W$". To do this, it would suffice to show that

- ☐ *(a)* All $X$ are $Z$, and some $Y$ are $W$.
- ☐ *(b)* All $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(c)* Some $Z$ are $X$, and some $Y$ are $W$.
- ☐ *(d)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(e)* Some $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(f)* Some $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(g)* All $X$ are $Z$, and all $Y$ are $W$.

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**Last Name**:. . . . . . . . . . . . . . . . . . . . . . . . . **First name**:. . . . . . . . . . . . . . . . . . . . . . . . . **Signature**:. . . . . . . . . . . . . . . . . . . . . . . . .

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**Mark the answers of the multiple-choice questions**

**(1)** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ Ⓕ Ⓖ
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**(1)** Suppose one wishes to prove that ”if some $X$ are $Y$, then some $Z$ are $W$”. To do this, it would suffice to show that

- ☐ *(a)* Some $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(b)* All $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(c)* Some $Z$ are $X$, and some $Y$ are $W$.
- ☐ *(d)* Some $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(e)* All $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(f)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(g)* All $X$ are $Z$, and some $Y$ are $W$.

**(2)** Let $X$ and $Y$ be statements. If we know that $X$ implies $Y$, then we can also conclude that

- ☐ *(a)* If $Y$ is false, then $X$ is false.
- ☐ *(b)* $Y$ cannot be false.
- ☐ *(c)* $X$ is true, and $Y$ is also true.
- ☐ *(d)* At least one of $X$ and $Y$ is true.
- ☐ *(e)* If $X$ is false, then $Y$ is false.
- ☐ *(f)* If $Y$ is true, then $X$ is true.
- ☐ *(g)* $X$ cannot be false.

**(3)** Let $X$ and $Y$ be statements. Which of the following strategies is NOT a valid way to show that ”$X \implies Y$”?

- ☐ *(a)* Assume that $X$ is false, and $Y$ is true, and deduce a contradiction.
- ☐ *(b)* Assume that $Y$ is false, and then use this to show that $X$ is false.
- ☐ *(c)* Show that some intermediate statement $Z \implies Y$, and then show that $X \implies Z$.
- ☐ *(d)* Show that either $X$ is false, or $Y$ is true, or both.
- ☐ *(e)* Assume that $X$ is true, and then use this to show that $Y$ is true.
- ☐ *(f)* Assume that $X$ is true, and $Y$ is false, and deduce a contradiction.
- ☐ *(g)* Show that $X$ implies some intermediate statement $Z$, and then show that $Z \implies Y$.

**(4)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that "$P(x)$ is true for some $x$ of type $X$", then we have to

- ☐ *(a)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.
- ☐ *(b)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.
- ☐ *(c)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type $X$.
- ☐ *(d)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(e)* Show that for every $x$ in $X$, $P(x)$ is false.
- ☐ *(f)* Show that there are no objects $x$ of type $X$.
- ☐ *(g)* Assume that $P(x)$ is true for every $x$ in $X$, and derive a contradiction.

**(5)** Let $X, Y, Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Y$ implies $Z$. If we also know that $Y$ is false, we can conclude that

- ☐ *(a)* $Z$ is false and $X$ implies $Z$. Correct Answer. $X$ is false and $X$ implies $Z$.
- ☐ *(b)* $X$ implies $Z$.
- ☐ *(c)* $X$ is false.
- ☐ *(d)* $Z$ is false.
- ☐ *(e)* None of the above conclusions can be drawn.
- ☐ *(f)* $X$ is false and $Z$ is false and $X$ implies $Z$.

**(6)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "At least one of $X$ and $Y$ are true", we need to show that

- ☐ *(a)* $X$ is true if and only if $Y$ is false.
- ☐ *(b)* $Y$ is false.
- ☐ *(c)* At least one of $X$ and $Y$ are false.
- ☐ *(d)* $X$ is false.
- ☐ *(e)* Exactly one of $X$ and $Y$ are false.
- ☐ *(f)* $X$ and $Y$ are both false.
- ☐ *(g)* $X$ does not imply $Y$, and $Y$ does not imply $X$.

**(7)** Let $P(n,m)$ be a property about two integers $n$ and $m$. If we want to prove that "For every integer $n$, there exists an integer $m$ such that $P(n,m)$ is true", then we should do the following:

- ☐ *(a)* Let $n$ and $m$ be arbitrary integers. Then show that $P(n,m)$ is true.
- ☐ *(b)* Let $n$ be an arbitrary integer. Then find an integer $m$ (possibly depending on $n$) such that $P(n,m)$ is true.
- ☐ *(c)* Show that whenever $P(n,m)$ is true, then $n$ and $m$ are integers.
- ☐ *(d)* Let $m$ be an arbitrary integer. Then find an integer $n$ (possibly depending on $m$) such that $P(n,m)$ is true.
- ☐ *(e)* Find an integer $n$ such that $P(n,m)$ is true for every integer $m$.
- ☐ *(f)* Find an integer $m$ such that $P(n,m)$ is true for every integer $n$.
- ☐ *(g)* Find an integer $n$ and an integer $m$ such that $P(n,m)$ is true.

**(8)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "Both $X$ and $Y$ are true", we need to show that

- ☐ *(a)* $Y$ is false.
- ☐ *(b)* Exactly one of $X$ and $Y$ are false.
- ☐ *(c)* At least one of $X$ and $Y$ are false.
- ☐ *(d)* $X$ is true if and only if $Y$ is false.
- ☐ *(e)* $X$ does not imply $Y$, and $Y$ does not imply $X$.
- ☐ *(f)* $X$ and $Y$ are both false.
- ☐ *(g)* $X$ is false.

**(9)** Suppose one wishes to prove that "if all $X$ are $Y$, then all $Z$ are $W$". To do this, it would suffice to show that

- ☐ *(a)* All $Y$ are $X$, and all $Z$ are $W$.
- ☐ *(b)* All $Z$ are $Y$, and all $X$ are $W$.
- ☐ *(c)* All $Y$ are $Z$, and all $W$ are $X$.
- ☐ *(d)* All $X$ are $Z$, and all $Y$ are $W$.
- ☐ *(e)* All $Z$ are $X$, and all $Y$ are $W$.
- ☐ *(f)* All $Z$ are $X$, and all $W$ are $Y$.
- ☐ *(g)* All $X$ are $Z$, and all $W$ are $Y$.

**(10)** Let $P(x)$ be a property about some object $x$ of type $X$. If we want to DISPROVE the claim that "$P(x)$ is true for all $x$ of type $X$", then we have to

- ☐ *(a)* Show that $P(x)$ being true does not necessarily imply that $x$ is of type $X$.
- ☐ *(b)* Show that there exists an $x$ of type $X$ for which $P(x)$ is false.
- ☐ *(c)* Show that for every $x$ in $X$, there is a $y$ not equal to $x$ for which $P(y)$ is true.
- ☐ *(d)* Assume there exists an $x$ of type $X$ for which $P(x)$ is true, and derive a contradiction.
- ☐ *(e)* Show that there exists an $x$ which is not of type $X$, but for which $P(x)$ is still true.
- ☐ *(f)* Show that there are no objects $x$ of type $X$.
- ☐ *(g)* Show that for every $x$ in $X$, $P(x)$ is false.

**(11)** Let $P(n, m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that "There exists an integer $n$ such that $P(n, m)$ is true for all integers $m$", then we need to prove that

- ☐ *(a)* For every integer $m$, there exists an integer $n$ such that $P(n, m)$ is false.
- ☐ *(b)* There exists an integer $m$ such that $P(n, m)$ is false for all integers $n$.
- ☐ *(c)* For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is false.
- ☐ *(d)* For every integer $n$, and every integer $m$, the property $P(n, m)$ is false.
- ☐ *(e)* If $P(n, m)$ is true, then $n$ and $m$ are not integers.
- ☐ *(f)* There exists integers $n$,$m$ such that $P(n, m)$ is false.
- ☐ *(g)* There exists an integer $n$ such that $P(n, m)$ is false for all integers $m$.

**(12)** Let $P(n, m)$ be a property about two integers $n$ and $m$. If we want to DISPROVE the claim that "For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is true", then we need to prove that

- ☐ *(a)* There exists integers $n$,$m$ such that $P(n, m)$ is false.
- ☐ *(b)* For every integer $n$, there exists an integer $m$ such that $P(n, m)$ is false.
- ☐ *(c)* There exists an integer $m$ such that $P(n, m)$ is false for all integers $n$.
- ☐ *(d)* For every integer $m$, there exists an integer $n$ such that $P(n, m)$ is false.
- ☐ *(e)* There exists an integer $n$ such that $P(n, m)$ is false for all integers $m$.
- ☐ *(f)* For every integer $n$, and every integer $m$, the property $P(n, m)$ is false.
- ☐ *(g)* If $P(n, m)$ is true, then $n$ and $m$ are not integers.

**(13)** Let $X$ and $Y$ be statements. If we want to DISPROVE the claim that "$X \implies Y$", we need to show that

- ☐ *(a)* $Y$ is true, but $X$ is false.
- ☐ *(b)* Exactly one of $X$ and $Y$ are false.
- ☐ *(c)* $Y$ is false.
- ☐ *(d)* $X$ and $Y$ are both false.
- ☐ *(e)* $X$ is false.
- ☐ *(f)* $X$ is true, but $Y$ is false.
- ☐ *(g)* At least one of $X$ and $Y$ is false.

**(14)** Let $X$, $Y$, $Z$ be statements. Suppose we know that $X$ implies $Y$, and that $Y$ implies $Z$. If we also know that $X$ is false, we can conclude that

- ☐ *(a)* $Z$ implies $X$.
- ☐ *(b)* $Z$ is false.
- ☐ *(c)* $Y$ is false and $Z$ implies $X$.
- ☐ *(d)* $Y$ is false.
- ☐ *(e)* $Y$ is false and $Z$ is false.
- ☐ *(f)* $Y$ is false, $Z$ is false and $Z$ implies $X$.
- ☐ *(g)* No conclusion can be drawn.