CONSTRUCTION METHODS FOR ALBERT ALGEBRAS

Albert algebras are exceptional simple cubic Jordan algebras. These algebras are of interest not only in their own right but also in view of their connection with exceptional groups. When working over fields, these algebras are fairly well understood, but distinctly less so over arbitrary commutative rings. Over fields, any Albert algebra can be obtained either by a first or by a second Tits construction. Already over domains this is no longer the case, even if one generalizes both constructions substantially as it was done independently by Achhammer and Parimala-Sridharan-Thakur. We present a third construction method for Albert algebras, which goes back to Allison and Faulkner and becomes interesting only for Azumaya algebras over rings.

Let R be a ring such that $1/2, 1/3 \in R$. Let A be an Azumaya algebra over R of constant rank 64, with a symplectic involution τ. Then $J = H(A, \tau)$ together with the multiplication $x \cdot y = \frac{1}{2}(xy + \tau(xy))$ is a Jordan algebra of degree 4 and rank 28. By providing the trace zero elements of J with a new multiplication, we turn them into an Albert algebra.