Course at University Milano-Bicocca

"Introduction to intersection theory in algebraic geometry"

Lecturer: Victor Lozovanu

Schedule: $14:00-17:00$, mostly on Fridays with some exceptions, starting beginning of February of total time of 40 hours.
Place: Room 3014, Department of Mathematics and Applications of University of Milano-Bicocca.

The Apollonius problem asks the number of circles tangent to three given circles in the plane. Bezout's theorem states that the number of points of intersection, when counted with the multiplicity, for two distinct algebraic curves in the projective plane is equal to the product of the degrees of the curves. A smooth cubic surface in the projective plane contains exactly 27 lines. All these problems are nowadays part of what's called intersection theory. This field sits at the heart of algebraic geometry. Dating back to Schubert, Chasles, Zeuthen, etc. in the 1800's, intersection theory is the study of intersections of cycles on an algebraic variety. Originally a tool for enumerative geometry, it has now grown beyond that, with applications and connections to Minimal Model Program, K-theory, Hodge theory, moduli spaces, Gromov-Witten invariants, quantum cohomology, and mirror symmetry.

The goal of these lectures is to introduce the audience to this circle of ideas with an emphasis on examples and computations. In some ways, the central value of this course is seeing algebraic geometry in action. This should be considered as an intermediate course in algebraic geometry. The first part, which time-wise will be the first month, will be presented in a basic manner and anyone involved in mathematics should be able to follow. The definition of a projective variety should be enough to understand the basic ideas. The second part of the course will cover topics that are only specific to algebraic geometry. Thus this part of the course requires a basic understanding of the modern algebraic geometry at the level of the first book of Shafarevich. We will barely touch the language of schemes, but make use of the cohomology theory as developed in Hartshorne. The theoretical part of the course is based on Fulton's book, whereas examples and applications will be inspired from the upcoming book of Eisenbud and Harris. Since this is a graduate students' course, I will focus partly on examples and applications of the main theory.

Prerequisites

- I. R. Shafarevich, *Basic Algebraic Geometry I: Varieties in Projective Space*, Third Edition, Springer, 2014.
- M. Reid, *Undergraduate Algebraic Geometry*, Cambridge University Press, 1988.
- A. Gathmann, *Algebraic Geometry*, online course. This course is a much better version of Hartshorne.
- R. Hartshorne, *Algebraic Geometry*, Grad. Texts in Math., Springer-Verlag, 1977.

Main References

- W. Fulton, *Intersection Theory*, Springer, 1998.
- D. Eisenbud and J. Harris, *3264 and all that: Intersection Theory in Algebraic Geometry*, upcoming book, online version.

Program:

Part 1: Main part of the course.

(1) Introduction to the course and presentation of three famous problems related to the subject.
(2) Bezout's theorem and applications. Theory of loops in real algebraic geometry.
(3) Cycles, Rational Equivalence an Chow groups.
(4) Cartier and Weil Divisors, intersections of divisors.
(5) Proper pushforward and flat pullback. Basic properties, functoriality, compatibility and exact sequences.
(6) Segre and Chern classes. Chow groups of vector bundles, grassmanians. Porteous formula.
(7) Segre classes of cones. Normal cones. Deformations of normal cones.
(8) Intersection multiplicity. Intersection ring of non-singular varieties..

Part 2: Advanced part of the course.

(9) Intersection products, refined Gysin homomorphisms.
(10) Excess intersections and blow-ups.
(11) Grothendieck-Riemann-Roch formula - an outline of the proof.

Grading: Anyone is invited to attend and participate in the course, especially due to its significance to many interesting classical and modern problems in algebraic geometry. As for the students who are willing to sign up officially to the course, they have to attend at least Part 1 of the course. This will cover around 30 hours. At the end I will allow students to present a project, or an interesting result connected to the main ideas of the course. They can choose by themselves the project or based on my advice. Sometimes there will be given sets of problems that I think are important to tackle alone. These are not compulsory, but many of them will help understand the material and the concepts in a better way. The other 10 hours of the course will be intended for more advanced part of intersection theory.

Schedule:

- Class 1: Wednesday, February 4th, 14:00–17:00, Room 3014.
- Class 2: Monday, February 16th, 14:00–17:00, Room 3014.
- Class 3–13: Each week on Friday, 1:00–17:00, Room 3014, with some exceptions due to holidays and unexpected things.