Programmi Cortona 15 agosto – 27 agosto 2010

General Loewner's deterministic theory

Docente : Filippo Bracci **–** Università di Roma Tor Vergata

Program: The generalized Loewner ODE and the generalized Loewner-Kufarev PDE. Schwarz's Lemma and the Poincarè distance in the unit disc. Automorphisms of the unit discs and their dynamics. Iteration of holomorphic self-maps of the unit disc. The Denjoy-Wolff theorem. Some intertwining models. Semigroups of holomorphic self-maps in the unit disc. The Berkson-Porta formula. Koenigs' functions and linearization of semigroups. Evolution families. Herglotz vector fields. Loewner chains as intertwining mappings. Loewner chains as families of Riemann mappings. The generalized Loewner ODE and the generalized Loewner-Kufarev PDE revised (and solved). Applications. Extremal problems: the Bieberbach conjeucture. Higher dimension generalizations: the unit ball. Complex hyperbolic manifolds.

Prerequisites: standard courses in one complex variable (with some advanced topics such as the Riemann mappings theorem, the Koebe distortion theorems) and a basic course in ODE. Rudiments of several complex variables and hyperbolic geometry might be of some help, although all used tools will be introduced.

Book: I. Graham, G. Kohr, "Geometric Function Theory in one and higher dimensions" Pure and Applied Mathematics, Marcel Dekker Inc., New York, 2003.

Suggested and further Readings:

1. Ch. Pommerenke, "Univalent Functions", Vandenhoeck & Ruprecht, Gottingen, 1975.

2. P.L. Duren, "Univalent Functions", Springer-Verlag, New York 1983.

3. S. Reich and D. Shoikhet, “Nonlinear semigroups, fixed points, and geometry of domains in Banach Spaces”, Imperial College Press, London, 2005.

4. F. Bracci, M. D. Contreras and S. Diaz-Madrigal, "Evolution families and the Loewner equation I: the unit disc", J. Reine Angew. Math., to appear (a copy is available on the ArXiV and on my webpage).

5. F. Bracci, M. D. Contreras and S. Diaz-Madrigal, "Evolution families and the Loewner equation II: complex hyperbolic manifolds", Math. Ann., 344 (2009), 947-962 (a copy is available on the ArXiV and on my webpage).

6. M. D. Contreras, S. Diaz-Madrigal and P. Gumenyuk, "Loewner chains in the unit disc", Rev. Mat. Iberoamericana, to appear (a copy is available on ArXiV).

7. L. Arosio, F. Bracci, H. Hamada, G. Kohr "An abstract approach to Loewner's chains", preprint 2010, available on ArXiV and on my webpage.

Filippo Bracci http://www.mat.uniroma2.it/~fbracci/

Docente : Robert Bauer – University of Illinois at Urbana

An introduction to the Schramm-Loewner evolution (SLE)

These lectures will provide an introduction to SLE. After discussing the circle of problems that eventually led Oded Schramm to introduce Stochastic Loewner Evolutions, their definition is given and basic properties derived. Next, some of the tools and methods used to study SLE and prove results such as Mandelbrot's conjecture are studied, and extensions to multiply connected domains considered. Finally, some more recent aspects concerning measures on loops in planar domains and in Riemann surfaces are discussed. The lecture plan is as follows:

1.Motivation: loop-erased random walk and iteration of conformal maps.
2. Loewner chains and the chordal Loewner equation: measuring the size of subsets of the half-plane.
3.Chordal SLE: Definition in the upper half-plane and in other domains, phases, and transience.
4.Restriction: Boundary perturbations, locality, and the restriction property.
5.Calculating with SLE: Schramm's formula.
6.Radial SLE: Definition in the unit disk and relation to chordal SLE.
7.Applications: Critical exponents and Mandelbrot's conjecture (Theorem).
8.SLE for multiply connected domains.
9.Measures on random loops obtained from SLE: Werner measure on Riemann surfaces.

In addition to the morning lectures, there will be afternoon sessions where students discuss and present assigned exercises and additional topics, or explore arguments from lecture in greater depth. The beginning part of the lectures is based on Wendelin Werner's *Random Planar Curves and Schramm-Loewner Evolutions* [We04]. A general reference for more background and more detailed proofs is Greg Lawler's *Conformally Invariant Processes in the Plane* [La05]. Further references for the second half of the course are given below. Prerequisites are a one semester graduate level course in probability.

<u>References</u>

[Ba09] Bauer, R., *A simple construction of Werner measure from chordal SLE(8/3)*, preprint, available at http://www.arXiv.org.

[BF08] Bauer, R., Friedrich, R., *On chordal and bilateral SLE in multiply connected domains*, Math. Zeitschrift **258**(2) (2008), 241-265.

[La05] Lawler, G., *Conformally invariant processes in the plane*, American Mathematical Society, Rhode Island, 2005.

[Sc01] Schramm, O., *A percolation formula*, Electron. Comm. Probab. **6** (2001), 115-120.

[We04] Werner, W., *Random Planar Curves and Schramm-Loewner Evolutions*, Lectures on Probability Theory and Statistics, Ecole d'Eté de Probabilités de Saint-Flour XXXII - 2002, ed. J. Picard, Springer, Heidelberg, 2004.

[We08] Werner, W., *The conformally invariant measure on self-avoiding loops*, J. Amer. Math. Soc. **21**(1) (2008), 137--169.