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## **An introduction to the Schramm-Loewner evolution (SLE)**

These lectures will provide an introduction to SLE. After discussing the circle of problems that eventually led Oded Schramm to introduce Stochastic Loewner Evolutions, their definition is given and basic properties derived. Next, some of the tools and methods used to study SLE and prove results such as Mandelbrot's conjecture are studied, and extensions to multiply connected domains considered. Finally, some more recent aspects concerning measures on loops in planar domains and in Riemann surfaces are discussed. The lecture plan is as follows:

1. Motivation: loop-erased random walk and iteration of conformal maps.
2. Loewner chains and the chordal Loewner equation: measuring the size of subsets of the half-plane.
3. Chordal SLE: Definition in the upper half-plane and in other domains, phases, and transience.
4. Restriction: Boundary perturbations, locality, and the restriction property.
5. Calculating with SLE: Schramm's formula.
6. Radial SLE: Definition in the unit disk and relation to chordal SLE.
7. Applications: Critical exponents and Mandelbrot's conjecture (Theorem).
8. SLE for multiply connected domains.
9. Measures on random loops obtained from SLE: Werner measure on Riemann surfaces.

In addition to the morning lectures, there will be afternoon sessions where students discuss and present assigned exercises and additional topics, or explore arguments from lecture in greater depth. The beginning part of the lectures is based on Wendelin Werner's *Random Planar Curves and Schramm-Loewner Evolutions* [We04]. A general reference for more background and more detailed proofs is Greg Lawler's *Conformally Invariant Processes in the Plane* [La05]. Further references for the second half of the course are given below. Prerequisites are a one semester graduate level course in probability.

### References

[Ba09] Bauer, R., *A simple construction of Werner measure from chordal SLE(8/3)*, preprint, available at <http://www.arXiv.org>.

[BF08] Bauer, R., Friedrich, R., *On chordal and bilateral SLE in multiply connected domains*, Math. Zeitschrift **258**(2) (2008), 241-265.

[La05] Lawler, G., *Conformally invariant processes in the plane*, American Mathematical Society, Rhode Island, 2005.

[Sc01] Schramm, O., *A percolation formula*, Electron. Comm. Probab. **6** (2001), 115-120.

[We04] Werner, W., *Random Planar Curves and Schramm-Loewner Evolutions*, Lectures on Probability Theory and Statistics, Ecole d'Eté de Probabilités de Saint-Flour XXXII - 2002, ed. J. Picard, Springer, Heidelberg, 2004.

[We08] Werner, W., *The conformally invariant measure on self-avoiding loops*, J. Amer. Math. Soc. **21**(1) (2008), 137--169.