Programmi Cortona 17 agosto – 26 agosto 2009

Geometric Aspects in Non-Linear Analysis

Docente : **Sun-Yung Alice Chang** - **Princeton University**

Some PDE in Conformal Geometry

Programma:

- In this course I will discuss some non-linear partial differential equations which arise in the study of prescribing curvature problems in conformal geometry. I will start with the problem of prescribing Gaussian curvature on compact surfaces; I will provide the background material of sharp Moser-Trudinger Sobolev inequality, then the result of uniformization theorem of compact surfaces. (e.g [1])
I will then cover the topic of ”Q-curvature” on manifolds, which can be viewed as a generalization of the Gaussian curvature. I will cover some basic PDE properties of the curvature, with emphasize on results on manifolds of dimension four. (e.g. [1], [2]) As an application, I will relate the integral of Q-curvature to the concept of ”renormalized volume” in conformal compact Einstein manifolds. ([2])
If time permits, I plan also to cover some relationship between Q-curvature and a class of symmetric functions of the Ricci tensor, and derive some local C2 estimates for this type of curvature tensor under conformal change of metrics. (e.g. [3])

Reference

(pdf files of [1] and [2] are available at www.math.princeton.edu/ chang)
[1] Sun-Yung A. Chang ”Nonlinear elliptic equations in conformal geometry”,
Nachdiplom lectures in Mathematics, ETH, Zurich, Springer-Birkh¨auser, 2004.

[2] Sun-Yung A. Chang and P. Yang ”The Q-curvature equations in conformal geometry”,to appear in volume 322 of Asterisque, GEOMETRIE DIFFERENTIELLE, PHYSIQUE MATHEMATIQUE, MATHEMATIQUES ET SOCIETY
(II), Volume en l’honneur de Jean Pierre Bourguignon (O. Hjazi, Editeur)

[3] Szu-Yu Sophie Chen ”Local Estimates for some fully non-linear equations”,
IMRN, no. 55, pp 3403-3425, 2005.

Docente : Andrea Malchiodi - S.I.S.S.A. Trieste

The Yamabe problem

Programma :

The course will cover many aspects of the Yamabe equation, starting from the most basic features and arriving to the least recent developments of the problem.We will discuss the realization of exremals for the criticalmSobolev inequality in R^n, and apply this study to derive existence for the Yamabe equation, exploiting its variartional structure and using Aubin's and Shoen's asymptotic expansions. Then we will turn to the compactness questions, using blow-up analysis and perturbative theory. Time permitting, we will also describe some features of thescalar curvature prescription problem on the standard sphere.

Text :
- T. Aubin, SOME NONLINEAR PROBLEMS IN RIEMANNIAN GEOMETRY, SPRINGER VERLAG, 1998.

References:

- M. Berti, A. Malchiodi, Non-compactness and multiplicity results for the Yamabe problem on S^n, J. Funct. Anal. 180, 2001.

- S. Brendle, F.C. Marques, Blow-up phenomena for the Yamabe equation II, J. Diff. Geom. 81, Number 2, 2009.

- J. Lee, T.Parker, The Yamabe problem, Bull. Amer. Math. Soc. 17 (1987) 37-91.

- M.A. Khuri, F.C. Marques and R.M. Schoen, A Compactness Theorem for the Yamabe Problem J. Diff. Geom 81, Number 1, 2009.