

FUNCTIONAL ANALYSIS

Alexander I. Bufetov

PROGRAMME

1. Metric Spaces. Examples. Completeness, Compactness, the Contraction Mapping Principle.
2. Normed Linear Spaces. Examples. The Space $C[0, 1]$. The Arzelà-Ascoli Theorem. The Peano Theorem.
3. Hilbert Spaces. The Representation Theorem of F.Riesz. Orthogonality. Strong and Weak topology. Compactness. Orthogonal Series and the Riesz-Fisher Theorem.
4. Banach Spaces. The Theorem of Hahn-Banach. The Theorem of Banach-Steinhaus. Banach's Closed Graph Theorem. Geometry of Locally Convex Spaces. The Krein-Milman Theorem.
5. Compact Operators. Hilbert-Schmidt Operators.
6. Representations of Compact Groups. The Peter-Weyl Theorem.
7. Spectrum of a Bounded Normal Operator. The Gelfand Representation.
8. Fourier Transforms. Convolutions. Generalized Functions. Sobolev Spaces and the Sobolev Embedding.
9. Linear Integral Equations. Fredholm's Theorems.
10. Elliptic Operators. The Spectrum of an Elliptic Operator.
11. The Hille-Yosida Theorem. Kolmogorov's Equations.
12. Representations of the Infinite Symmetric and the Infinite Unitary Group (following Vershik-Kerov-Olshanski-Okounkov-Borodin).

Textbooks.

1. Elements of the Theory of Functions and Functional Analysis,
by A. N. Kolmogorov, S. V. Fomin.
Dover Publications 1999; ISBN-10: 0486406830; ISBN-13: 978-0486406831.
2. Functional Analysis, by K. Yosida.
Springer Verlag 2003; ISBN-10: 3540586547; ISBN-13: 978-3540586548.