STOCHASTIC NUMERICAL METHODS OF FINANCE

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Pricing and hedging without martingale measures

Description
This part of the course complements that of Prof. Platen in the sense that it concerns some fundamental aspects of pricing and hedging that result from the following :

• In classical Mathematical finance the absence of arbitrage and the existence of an equivalent martingale measure (EMM) are regarded as essentially equivalent concepts.

• Real markets may however allow for some anomalies; in particular one may have that
a) stock price bubbles occur; in other words, the discounted price processes are strict local martingales under a risk neutral measure. This phenomenon is still consistent with classical no-arbitrage theory, but some of the classical results do not hold anymore;
b) the candidate martingale density process may be a strict local martingale and so even an EMM does not exist anymore. This is a more severe anomaly and most of the usual results from Mathematical finance then break down.

Objectives

• We shall concentrate on the second anomaly in b) and show that, even in the absence of an EMM, there is still a meaningful way to proceed in order to solve the crucial problems of pricing and hedging of contingent claims, in particular under market completeness (the latter can be defined also under absence of an EMM)

• We plan to discuss these issues along three approaches that are essentially equivalent in the sense that they all lead to the so-called real world pricing formula, which corresponds to the standard valuation formulas, where the numeraire is now the GOP and the pricing measure is the physical measure. These approaches are:
i) The "upper hedging approach". Main reference [1], see also [3].
ii) The "Benchmark approach". Main reference [4] and Chapter 3 in [5]. The equivalence between this approach and the previous one is discussed in [6].
iii) The "utility indifference" approach in the context of the Benchmark approach. Main references again [4] and Chapter 3 in [5].

• The lectures concern the above topics as outlined in the survey article [2]. Students interested in giving a seminar on these topics may have a preliminary look at Chapter 3 in [5].

References

[1] Fernholz, R. & Karatzas, I. (2009), Stochastic Portfolio Theory: an Overview, in: Bensoussan, A. & Zhang, Q. (eds.): Mathematical Modeling and Numerical Methods in Finance, Handbook of Numerical Analysis, vol. XV, North-Holland, Oxford.
[2] Fontana, C. (2010), Diffusion-based models for financial markets without martingale measures: an overview. Preprint.
[3] Karatzas, I. & Kardaras, K. (2007), The Num´eraire Portfolio in Semimartingale Financial Models, Finance and Stochastics, 11: 447-493.
[4] Platen, E. & Heath, D. (2006), A Benchmark Approach to Quantitative Finance, Springer, Berlin - Heidelberg.
[5] Platen, E. & Bruti-Liberati, N. (2010) Numerical Solution of Stochastic Differential Equations with Jumps in Finance. Stochastic Modelling and Applied Probability Volume 64, Springer.
[6] Galesso, G. & Runggaldier, W.J. (2010), Pricing Without Equivalent Martingale Measures under Complete and Incomplete Observation, In: Chiarella, C. & Novikov, A. (eds.), Contemporary Quantitative Finance: Essays in Honour of Eckhard Platen, Springer.