Università di Milano–Bicocca
Quaderni di Matematica

Problem-based learning and teacher training in mathematics

Marina Cazzola

Quaderno n. 8/2008 (arxiv:0805.477v1)

Stampato nel mese di maggio 2008
presso il Dipartimento di Matematica e Applicazioni,
Università degli Studi di Milano-Bicocca, via R. Cozzi 53, 20125 Milano, ITALIA.

Disponibile in formato elettronico sul sito www.matapp.unimib.it.
Segreteria di redazione: Ada Osmetti - Giuseppina Cogliandro
tel.: +39 02 6448 5755-5758 fax: +39 02 6448 5705

Esemplare fuori commercio per il deposito legale agli effetti della Legge 15 aprile 2004 n.106.

PROBLEM-BASED LEARNING AND TEACHER TRAINING IN MATHEMATICS

MARINA CAZZOLA

ABSTRACT. Problem-based learning (PBL) is a constructivist learner-centered instructional approach based on the analysis, resolution and discussion of a given problem. It can be applied to any subject, indeed it is especially useful for the teaching of mathematics.

When compared to "traditional" teaching, the PBL approach requires increased responsibility for the teachers (in addition to the presentation of mathematical knowledge, they need to engage students in gathering information and using their knowledge to solve given problems). It thus become crucial that the future teachers become aware of its effectiveness. One of the main obstacle to this awareness lies usually on the fact that future teachers did not find this methodology in their own pre-service training. In this paper we will describe the attempt to introduce PBL in University courses so to have future maths teacher "experience mathematics" themselves.

1. PROBLEM-SOLVING, PBL, AND MATHEMATICS

> *A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. Such experiences at a susceptible age may create a taste for mental work and leave their imprint on mind and character for a lifetime.*" (Polya, 1945, p. v)

We like to start with this old quote from Polya, as these few line contain a meaningful view of mathematics and of mathematics education. Indeed, in Polya's view the act of "solving problems" is the key to access *true* mathematical knowledge, and teaching effectively mathematics means having students become competent problem-solvers. This basic idea has been gaining more and more ground; simply put Polya is recognised as the founder of mathematical problem solving; his book "*How to Solve It* planted the seed of the problem-solving 'movement' that flowered in the 1980s" (Schoenfeld, 1992, p. 352). Basically, in Polya's view, teaching mathematics should not focus on the transmission of *information*, but rather on the development of the *ability to use this information*, or, in Polya's words, on the transmission of *know-how*: "Our knowledge about any subject consists of information and know-how. Know-how is ability to use information; of course, there is no know-how without some independent thinking, originality, and creativity. Know-how in mathematics is the ability to do problems, to find proofs, to criticise arguments, to use mathematical language with some fluency, to recognise mathematical concepts in concrete situation" (Polya, 1962, vol. 2, p. 112). This "old" point of view is far from getting out of fashion, and the need of the kind of competencies pointed out by Polya arise from the 'real world'. When asked to speak at the latest Congress of the *Unione Matematica Italiana*, employers stated that in their industries the feel the need for mathematicians that "can solve ill-structured problems, that are willing to look for flexible solutions, and are able to apply these solutions to parts of the problem that can arise in subsequent steps" (Barberis, 2007). They

Date: Draft: 2008/05/28.

suggested that mathematics instruction should "appoint students to solve not only classical problems, but also problems that are badly formulated, not well defined problems, and possibly with some contradictory data" (Barberis, 2007).
Actually research in education share what we called Polya's view, especially with the rise of the "new" socio-constructivist movement. Inspired by these new pedagogical ideas, school reforms were promoted. A clean example is the US "reform" stimulated by the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics. These standards promoted a view of mathematics that can be summarised by the following three points: mathematics instruction should lead students into

- "Seeking solutions, not just memorising procedures;
- Exploring patterns, not just memorising formulas;
- Formulating conjectures, not just doing exercises." (National Research Council, 1989)

The standards suggested an "innovative" model of instruction that should give students "the opportunity to study mathematics as an exploratory, dynamic, evolving discipline, rather than as a rigid, absolute, closed body of laws to be memorised" (National Research Council, 1989).
What so far appears as a clear picture of a neat evolution of the teaching practice, switching from a traditional model, based on teachers *transmitting information* to the pupils, to an innovative one, based on the *construction of know-how*, does not reflect the truth. Indeed, the reform we mentioned gave rise to a big controversy well known as the "Math Wars". A discussion full of hanger was held on any kind of media (local, regional and national newspaper and magazines, radio, national television), but often the arguments given did not rely on solid grounds (e.g. see (Schoenfeld, 2004)). Nevertheless the whole story shows that the dispute about tradition and innovation is still far from being settled down. The resistance to new methodologies could be caused by an unclear view of what "problem-solving" means: "one might infer, that there is general acceptance of the idea that the primary goal of mathematics instruction should be to have students become competent problem solvers. Yet, given the multiple interpretations of the term, the goal is hardly clear. Equally unclear is the role that problem-solving, once adequately characterised, should play in the larger context of school mathematics. What are the goals for mathematics instruction, and how does problem-solving fit within those goals?" (Schoenfeld, 1992, p. 334). Moreover "'problems' and 'problem-solving' have had multiple and often contradictory meanings through the years—a fact that makes interpretation of the literature difficult" (Schoenfeld, 1992, p. 337). In his interesting review, Schoenfeld collects a list of often contradictory "goals for courses that were identified by respondents as 'problem-solving' courses:

- to train students to 'think creatively' and/or 'develop their problem-solving ability' (usually with focus on heuristic strategies);
- to prepare students for problem competitions such as the Putnam examinations or national or international Olympiads;
- to provide potential teachers with instruction in a narrow band of heuristic strategies;
- to learn standard techniques in particular domains, most frequently in mathematical modeling;
- so provide a new approach to remedial mathematics (basic skill) or to try to induce 'critical thinking' or 'analytical reasoning' skills." (Schoenfeld, 1992, p. 337)

In other words, the term problem-solving can be misleading. On top of that we should recall that often the words "innovation", "mathematical discovery", and "problem-solving" were used as mere *slogan system*, rather than with full knowledge. We get an interesting point of view by Freudenthal: "'problem-solving' and 'learn by discovery' are now fashionable expression. I never liked them, neither at the beginning, when they were used as 'slogan', nor now, as I can see examples of. 'Problem-solving': that is solve the problem

given by the teacher, or by the author of the textbook, or by the researcher, using the tricks that they have in their mind [...] ‘learn by discovery’ that is find what has been hidden: like you do with Easter eggs” (Freudenthal, 1991)[1].
When we refer to problem-solving we mean exactly Polya’s view we started from, that is “learning to grapple with new and unfamiliar tasks when the relevant solution methods (even if only partly mastered) are not known” (Schoenfeld, 1992, p. 354), and we think that mathematics instruction should develop the students’ “ability to solve problems in new contexts or to solve problems that differ from the ones one has been trained to solve” (Schoenfeld, 2004, p. 262). In other words we would like to focus our attention to the use of problems as a tool to stimulate learning, that is what is known as *Problem-based learning* or PBL.

1.1. **Problem-based learning (PBL).** The socio-constructivist view of teaching and learning mathematics gives recognition and value to new instructional strategies in which students are able to learn mathematics by personally and socially constructing mathematical knowledge. For a neat definition of PBL we refer to (Savery, 2006) according to which “Problem-based learning (PBL) is an instructional approach that has been used successfully for over 30 years and continues to gain acceptance in multiple disciplines. It is an instructional (and curricular) learner-centered approach that empowers learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem.”
Typically a PBL session follows these steps:

- pupils are given a problem;
- they discuss the problem and/or work on the problem on their own and/or in small groups, collecting information useful to solve the problem;
- all the pupils gather together to compare findings and/or discuss conclusions; new problems could arise from this discussion, in this case
- pupils go back to work on the new problems, and the cycle starts again.

Variation can occur, but there are a few key elements one should take care. We will briefly report them here.

1.1.1. *Problems.* “Critical to the success of the approach is the selection of ill-structured problems (often interdisciplinary)” (Savery, 2006, p. 12); “cognitive research and practical experience with PBL have made important strides in identifying the characteristics of a good problem” (Hmelo-Silver, 2004, p. 244). To be more precise in order “to foster flexible thinking, problems need to be complex, ill-structured, and open-ended; to support intrinsic motivation, they must also be realistic and resonate with the students’ experiences. A good problem affords feedback that allows students to evaluate the effectiveness of their knowledge, reasoning, and learning strategies. The problems should also promote conjecture and argumentation. Problem solutions should be complex enough to require many interrelated pieces and should motivate the students’ need to know and learn” (Hmelo-Silver, 2004, p. 244). While we share the conviction that problems need to be “realistic”, and that pupils will find no interest in problems dealing with a “cooking pot with the shape of a right prism with base a regular triangle with sizes 1 meter long” (Dedò, 2001), in our experience the connection of the problems with students’ experiences is not so crucial: if the problems contains some “good maths”, the intellectual challenge is the mean to get students’ attention (e.g. see examples in (Bonaiti et al, 2005), (Cazzola, 2007), (Caronnni et al, 2007), and (Bolondi, 2005))

[1]Our source is in fact Freudenthal (1994).

1.1.2. *Group work.* Collaboration in essential. “As students generate hypotheses and defend them to others in their group, they publicly articulate their current state of understanding, enhancing knowledge construction and setting the stage for future learning” (Hmelo-Silver, 2004, p. 244). According to our experience, the group work also motivate pupils, and in the group they find support for dealing with difficult tasks they would not be able to tackle by themselves.

1.1.3. *General discussion.* “A closing analysis of what has been learned from work with the problem and a discussion of what concepts and principles have been learned are essential” (Savery, 2006, p. 14). Indeed a general discussion in which all the groups gather together to compare findings and discuss solution is the step that really consolidate the learning. The effectiveness of the general discussion rely upon the fact that pupils know what is going on as they have been working on the problem, “given that PBL is a very engaging, motivating and involving form of experiential learning, learners are often very close to the immediate details of the problem and the proposed solution” (Savery, 2006, p. 14).

1.1.4. *What do students learn?* “Research indicates that classroom instruction, which tend to focus almost exclusively on the knowledge base, deprives students of problem-solving knowledge” (Schoenfeld, 2004, p. 263), while researches show the effectiveness of a PBL approach to improve students’ problem-solving abilities (e.g. see Hmelo-Silver (2004); Mergendoller et al (2006)). We must stress the fact that a PBL approach helps pupils to get abilities that go beyond the mathematical contents they have been assigned. “Students develop flexible knowledge, effective problem-solving skills, SDL skill, effective collaboration skills and intrinsic motivation”, furthermore they develop “flexible understanding and lifelong learning skills” Hmelo-Silver (2004). By design, PBL helps students

- “construct an extensive and flexible knowledge base;
- develop effective problem-solving skills;
- develop self-directed, lifelong learning skills;
- become effective collaborators; and
- become intrisecally motivated to learn.” Hmelo-Silver (2004)

No need to stress the fact that these essential skills are what students will need in the world after schools.

1.1.5. *Teacher roles and teacher training.* With a PBL approach the teacher acts as a facilitator, guiding the the learning process and conducting the final debriefing at the conclusion of the learning experience. Also the teacher needs to plan and set up the activities. Although research collects data proving the effectiveness of a PBL approach, such an approach is not so well established in the teaching practice. There are of course criticism about PBL. A clear disadvantage to the use of PBL is that it can be really time consuming. Moreover “Many lecturers have expressed concern about whether sufficient knowledge can be conveyed through a PBL format and whether students have sufficient prior experience to be able to benefit from the problem-solving situation. Another concern is that when students are initially confronted with this approach many are suspicious about its value, particularly if they have previously been used to teacher-centered approaches” (Taplin and Chan, 2001, p. 287). Although an explanation of the resistance towards this kind of approach can be found in the teachers beliefs, that we will discuss in the following section, the lack of training in this kind of activities deters teachers from choosing such a approach: “one barrier to using PBL in more diverse settings is the lack of a sufficient number of skilled facilitators in many settings” (Hmelo-Silver, 2004, p. 261). Models for teacher training in this kind of activity typically suggest these kind of actions (e.g. see (LeBlanc, 1982), but note that he actually focused on *problem-solving*):

- solving problems: teachers should be given some problems to solve, class interaction should focus also on how a problem might be solved rather than on the actual solution;
- having children solve problems: teachers should learn to monitor their pupil actions and reactions;
- compiling a list of problems: teachers should compile a list of problems for future use.

According to LeBlanc (1982), "teachers report that their own attitudes toward problem-solving and instruction have been changed by this training and that their students *can* and *do* solve problems and enjoy the process". He concludes aiming researches would follow focusing on the analysis of the changes this kind of training induce in teachers attitudes.

1.2. **Teachers' beliefs.** "Teachers hold well-articulated educational beliefs that in turn shape instructional practice" (Handal, 2003, p. 47), "teacher's sense of the mathematical enterprise determines the nature of the classroom environment that the teacher creates".(Schoenfeld, 1992, p. 359). Handal (2003) review the literature on teachers' beliefs, especially when they face the choice of adopting an innovative teaching practice. What emerges is that teacher beliefs "act as a filter through which teachers make their decision rather than just relying on their pedagogical knowledge or curriculum guidelines" (Handal, 2003); in spite of having had "instruction about up-to-date methods of teaching mathematics, [teachers] often revert to teaching styles similar to those of their own teachers; they show little or no change in their conceptions of mathematics teaching despite their method courses" Taplin and Chan (2001). Indeed this reversion to "traditional teaching" is often due to strong beliefs that teachers acquired "symbiotically from their former mathematics school teachers after sitting and observing classroom lessons for literally thousands of hours throughout their past schooling", "once acquired, teachers' beliefs are eventually reproduced in classroom instruction", and "there is some evidence that, in some cases, teacher education programs are so busy concentrating on imparting pedagogical knowledge that little consideration is given to modifying these beliefs".
To change this trend, teacher training practise should switch from a traditional model to an innovative one, that is prospective teachers should be taught in a manner similar to how they are to teach.

2. The experience

If we start from the assumption that "teachers teach the way they have been taught" Handal (2003) (quoting Frank, 1990) it is not surprising that once graduated these pre-service teachers hold the belief that frontal lecturing is an essential part of teaching. Even if the lectures they attended at university were all devoted to explaining how active learner-centered methodology are much more effective than traditional lectures. Indeed studies show that "a large population of teachers still believe that teaching and learning mathematics is more effective in the traditional model" (Handal, 2003, p. 50).
The experience is part of a "traditional" pre-service teacher training university course. When we say traditional, we mean that most of the teaching is carried on through frontal lectures addressed to a large audience (about 100 people). To be honest it is exactly the situation in which what we teach about teaching is completely different from the way we teach. We would not be surprised if our students addressed us the objection "you are telling me beautiful things, but if I do not see you using such methodology, why should I believe it is worth the effort?".
Our intention in carrying out this experience was to explore the extent to which the pre-service teacher developed beliefs in favor of the use of a PBL methodology and, at the same time, developped positive attitudes toward creative mathematical activities.

2.1. **Lectures.** The course we used for the project was the 30-hours module "teaching mathematics"[2], at the University of Milano-Bicocca (Italy). The experience involved students in their third year[3]. In their previous career students already attended two 30-hours module on mathematical contents (whose name would sound "Elements of mathematics"[4]) focusing on arithmetic (in their first year) and on Euclidean geometry (in their second year). For a description of the content of these courses and of the whole pre-service teacher training university course we refer to (Cazzola, 2004). Although devoted to pre-service teachers, these courses were attended by some in-service teachers too.
Although, as we said, we carried on a standard university course, we attempt as much as possibile to introduce dialogue in the lectures, but the dimension of the class makes things difficult. Anyway, the Faculty is open minded and experiment of innovative teacher methodologies are welcome. The pre-service teachers attend standard lecture and a series of pedagogical laboratories covering almost all curricular subjects. Moreover our University hosts the Center matematita, Interuniversity research center for the communication and informal learning of mathematics.
What we try to describe here is a first attempt to adopt PBL methodologies to the standard university lessons. In doing so we were once again inspired by Polya and kept in mind his experience "all the classes I have given to mathematics teachers were intended to be methods courses to some extent. The name of the class mentioned some subject matter, and the time was actually divided between that subject matter and methods: perhaps nine tenths for subject matter and one tenth for methods. If possible, the class was conducted in dialogue form. Some methodical remarks were injected incidentally, by myself or by the audience. Yet the derivation of a fact or the solution of a problem was almost regularly followed by a short discussion of its pedagogical implications. 'Could you use this in your classes?' I asked the audience. 'At which stage of the curriculum could you use it? Which point needs particular care? How would you try to get it across?' And questions of this nature (appropriately specified) were regularly proposed also in examination papers. My main work, however, was to choose such problems [. . .] as would illustrate strikingly some pattern of teaching" (Polya, 1962, vol. 2, p. 115).
However, university lesson are subject to many bonds: time constraint, large classes, syllabus to cover. Nevertheless these are exactly the obstacles that future teachers must deal with and we have the chance to show our students that such obstacles can be overridden. If we give up, why should we expect our students to persist? Moreover as PBL is a methodology particularly effective for mathematics learning, it should be our first choice, as we have to teach mathematical content. When future teacher experience the methodology themselves they get the perception of the effectiveness of PBL as they realize that in this way they learn some good mathematics.

2.2. **PBL sessions and problems.** During 2007, part of the curricular time was devoted to two PBL session in which students were presented with a series of problems. The two session lasted 4 hours, two of which were spent in small group work on the problems, while the othe two were devoted to the general discussion. To some extent, our experience is similar to the one described in (Taplin and Chan, 2001), with the main difference that they gave the students pedagogical problems, while our basic idea was to transmit at the same time both pedagogical and mathematical knowledge to our students. We stress once again the fact that by giving them mathematical problems, students in a PBL environment gain beliefs in favour of a PBL approach, as they see that with this setting they manage to grasp the argument given. But we should not forget that we also need to give the pre-service teachers a solid background, as if "the teacher has had no experience in creative work of

[2]*Didattica della matematica*
[3]The pre-service teacher training university degree course is a four years program.
[4]*Istituzioni di matematiche*

Instruction. During this session you will be given a list of problems, problems you will have to solve during the first two hours working in group; the following time will be devoted to a general discussion with the other groups in order to confront your solutions and, of course, the methods you used to get your solutions. Be sure that every group has a member attending the general discussion. At the end of the first two hours hand in to me your written solution with the list of the group members. The problems could be difficult: work together and seek confrontation in your group. In the final discussion any member of the group should be able to explain the group findings/thesis. And if the group has more than one thesis it will be useful to write all of them down on the answer sheet and report all of them during the final discussion. You will be given many problems: today you are not required to solve them all, but it is important that when you work on a problem you have full concentration. So, make a choiche within your group on which problem tackle and do not start working on a new one unless you have finished with the previous one. Keep in mind that… these problems could be exam questions. Good work!

TABLE 1. Instruction for the students

some sort, how will he be able to inspire, to lead, to help, or even to recognize the creative activity of his students? A teacher who acquired whatever he knows in mathematics purely receptively can hardly promote the active learning of his students. A teacher who never had a bright idea in his life will probably reprimand a student who has one instead of encouraging him" (Polya, 1962, vol. 2, p. 112). Finally, teachers using a PBL approach need to have a good content knowledge of the subject they are teaching as they have to deal with students active learning, that often leads to conjecturing, proposing solution and, sometimes, to making up unexpected questions.
Lectures preceding the PBL sessions were devoted to illustrating the PBL approach. After the sessions, students were supposed to compile a list of problems as part of the final examination.
The PBL sessions were introduced by the instructions given in Table 1. The students were given the problems and asked to work in groups of four.
Instruction were ment as a guidance on how the session was supposed to work out, but also aimed at motivating the students. In this sense one should read the warning about difficult problems and the final remark "Keep in mind that… these problems could be exam questions".
In each of the two PBL session students were given 3 problems and asked to make a choice. The choice of giving so many problems was dicated by the time constraints and the idea that in this way we would have covered a larger amount of mathematical contents. However it is a choice that has to be reviewed and we will discuss later in this paper.

2.2.1. *Problems.* Problems were chosen in order to cover part of the syllabus, and "teaching probability" is a relevant part of it. Also combinatorics is propedeutic to probability and gives a lot of examples that allow manipulation (and therefore can give pre-service teachers useful ideas on activities that can be proposed to young children).

2.3. **The questionnaire.** The purpose of the questionnaire was to have the students revise the experience, particularly testing the following aspects

- check students' ability to recognize strengths and weaknesses of the activity;

(1) Given a set A and a set B, how many functions from A to B?
   (a) Let A be the set consisting of the letters a, b, c and B be the set consisting of the numbers $1, 2$. List all the functions from A to B.
   (b) It is possibile to tell the number of functions of the previous question without actually writing down the list of all such functions?
   (c) Given a set A with n elements and a set B with m elements, how many functions from A to B is it possible to write down?

(2) Given a set A or order n, how many subsets does A has?
   (a) Given a set A of order 5, how many subsets with 2 elements does A have? And how many with 3 elements? And 4? And ...?
   (b) Give a set A of order n, how many subsets with 2 elements does A have? And how many with 3 elements? And how many with $n-1$ elements?
   (c) Durung on of the lecures we dicovered the "magic" of Tartaglia's triangle

$$\begin{array}{ccccccccccccccc}
 & & & & & & & 1 & & & & & & & \\
 & & & & & & 1 & & 1 & & & & & & \\
 & & & & & 1 & & 2 & & 1 & & & & & \\
 & & & & 1 & & 3 & & 3 & & 1 & & & & \\
 & & & 1 & & 4 & & 6 & & 4 & & 1 & & & \\
 & & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & & \\
 & 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 & \\
1 & & 7 & & 21 & & 35 & & 35 & & 21 & & 7 & & 1
\end{array}$$

   Can you justify the fact that such a triangle contains exactly the numbers we are looking for?
   (d) What is the sum of each row of the Tartaglia's triangle? Can you explain such a result?

(3) Given two sets A and B we say that A has the same cardinality as B if there exists a bijection from A onto B.
   (a) The relation "to have the same cardinality as" is an equivalence relation?
   (b) A set is *countable* if it has the same cardinality as the set $\mathbb{N}$ of natural numbers. Is $\mathbb{Z}$, the set of integers, countable?
   (c) Are the subsets of $\mathbb{N}$ countable?
   (d) During standard lectures we proved that the set

$$I = \{x \in \mathbb{R} \mid 0 < x < 1\}$$

   is not countable (i.e. is uncountable). Do you think that the cartesian product $I \times I$ has the same cardinality as I? Does it have a *larger* or a *smaller* cardinality, whatever that means?

TABLE 2. Problems for the first PBL session

- monitor beliefs about the PBL approach, in particular get an idea if they had developed a willingness to implement a PBL teaching model;
- monitor beliefs about the role that universities can have in their future profession of teachers.

Note that question 1 was meant to be ambiguous, not specifying whether the "things learned" should be read as regarding mathematical or pedagogical contents. This was done in order to check the student perception of the experience.
Question about *what did work* and *what did not work* just wanted to stimulate in the students the reflection on the fact that the experience could be improved. Also, note that most of the question are sort of asked twice: clearly question 2 and question 4 are related, as are question 3 and question 5.

(1) A bag contains 4 red balls and 3 yellow balls.
   (a) We draw 2 balls out of the bag, one after the other, putting the drawn ball back in the bag. What is the probability that the two drawn balls are both red? That they are both yellow? That they are of different colours?
   (b) We draw 2 balls out of the bag, one after the other, without replacing the drawn ball back in the bag. What is the probability that the two drawn balls are both red? That they are both yellow? That they are of different colours?
   (c) We draw 2 balls together, at the same time, out of the bag. What is the probability that the two drawn balls are both red? That they are both yellow? That they are of different colours?

(2) (a) I was born in a non-leap year. Piero is born in a non-leap year too. What is the probability that Piero and I share our birtday?
   (b) In a group of 3 people, what is the probability that at least two of them share their birthday? And in a group of 4 people?
   (c) How many people should be present (??) in this room to be sure that at least two of them share their birthday?
   (d) Are you willing to bet that at least two people in this room share their birthday?

(3) There are three closed doors. One of these hides a Ferrari, the other two just a box of chocolates.
   You can choose one of the doors and take away what's hidden behind it. When you choose a door, a friend opens one of the remaining doors, showing the box of chocolates hiding behind it.
   At this point you can make a new choice. You can either keep the door you have chosen first or change it. What would you do?
   Why?

TABLE 3. Problems for the second PBL session

Question 8 was meant to test the beliefs of the students and their awareness of possible difficulties in actually adopting a PBL approach, and test whether university is seen as a resource by prospective teachers.
Some of the pre-service teachers were interwied to clarify part of their answer in the questionnaire
The questionnaire was submitted to the students shortly after the second pbl session, before making any comment on the experience with them. 94 questionnaire were collected and analyzed.

3. RESULTS

3.1. **Analysis.** We will focus on the three different aspect we had in mind to monitor.

3.1.1. *Ability to recognize strengths and weaknesses of the experience.* With the questionnaire we tested the students' attitude towards the experience. In particular, the activity strengths are highlighted in Table 5 that contains categorized answers to questions 1 and 2 In particular, looking at the answer given to question 1, the studens include pedagogical contents too. It is interesting to compare Table 5 with Table 7 that contains categorized answers to the open part of question 4. What strikes us the most is the fact that there is in the students a clear reference to "group-work" rather than PBL approach (even if during their course of studies they have many pedagogical content courses explaning distinctions). In the interview some students admitted that when they answered the questionnaire they hadn't studied yet. We think it is significat that some stutents feel the needs to express their persona satisfaction with the experienze in answering question 1, althought we do not

QUESTIONNAIRE

On March 21 and March 28 we had two problem sessions in which we experimented a *problem-based learning* methodology.

(1) What do you think you have learned in these sessions? ....
(2) What *did work* during the two lectures?
  Session of March 21 ....
  Session of March 28 ....
  Both session ....
(3) What *did not work*?
  Session of March 21 ....
  Session of March 28 ....
  Both session ....
(4) Are you likely to use a methodology of the type we experimented for the mathematics lessons in your future teaching profession? yes ☐ no ☐
  Why? ....
  And for the other curricular subjects? yes ☐ no ☐
  Why? ....
(5) If you answered "yes" to the previous question, would you set up your lectures in a way different from the one we experimented? yes ☐ no ☐
  If yes, which changes? ....
(6) If you answered "no" to question 4, would you be willing to use such a practice only if you could modify it? yes ☐ no ☐
  If yes, how? ....
(7) If you answered "yes" to question 4, what part of the mathematical curricular time would you devote to a *problem-based learning* approach? ....
(8) What kind of support do you think you will need in order to use a *problem-based learning* approach? ....
  From colleagues? ....
  From the school principal ....
  From the University? ....
  From others? ....
(9) If you have any comments, write them here. ....

TABLE 4. The questionnaire

want to open a discussion on "affects". Looking at weaknesses of the experience, Table 6 collects answer to question 3, while Table 8 contains answers to question 5 and question 6 (some students answered to question 6 even if they were not supposed to do so). It could be useful to compare the two tables. In our opinion, criticism listed in Table 6 are truly due to a missconduction of the experience. We take the blame for this, but wish to defer comments later in this paper (the real surprise come from the 3 answering "everything worked smoothly"). By the category "negative comments due to expectations tipical of a 'traditional' didactical model" we mean answer of the type "we were given no textbooks to study ahead", "we were not able to solve the problem because we hadn't studied", in other words clearly focusing on a lack of *information* as opposed to Polya's *know-how*. While I accept most of the criticism contained in table 6, I had the chance to interview the student who answered "there was only a photocopy for each group", and explain her that that this was exactly the trick that helped to get the group members work together.

3.1.2. *Willingness to use a pbl approach.* In order to test the willingness to use a pbl approach I do not think that the yes/no answer to question 4 is somehow significant: the

express satisfaction with the experience	12
express changing in attitude towards 'doing' mathematics (I used to do mathematics on my own)	4
express negative emotions	1
learned to work in group, learned importance of group work to create a comfortable atmosphere	80
recognize the two phases of a PBL (both group work and general discussion, confrontation among groups)	34
new beliefs on the methodology ("we built up something that cannot be found in books", "you can enjoy learning")	17
learned to deal with time	11
you need good problems to use PBL	8
processes are more important than the mere results ("we got wrong result, but we learned some good maths anyway")	7
learned some mathematical content (name mathematical contents without focusing on any in particular)	25
new beliefs on mathematics ("reasoning is a crucial part of mathematics", "formulas learn by hearth worth nothing", "you can try to guess first")	22
learned a particular mathematical content	7

TABLE 5. Analysis of results: things learned

feeling of a lack of a conclusion, mostly due to the general discussion	37
logistics (e.g. small lectureroom)	31
negative comments due to expectations tipical of a 'traditional' didactical model	21
criticism about the groups (the chosen groups did not work out)	18
too many problems, not enough time ("all the problems were interesting, but we had not enough time to deal with them")	16
too long, too tiring	4
no tutoring	3
everithing worked smoothly	3
did not reach the PBL goals ("did not feel like I was part of the group discoveries", did not commit to the group)	2
there was only a photocopy for each group	1

TABLE 6. Analysis of results: things that did not work

importance of group work	32
this metodology makes mathematics interesting, accattivante, coinvolgente, piacevole, less boaring, one understand he can make it; you give pupils a differnt view of mathematics	24
in order to learn you need to cohoperate and to confront other people's views	20
this methodology is effective to produce long life learning	18
in mathematics you need to put theory into practice	11
with this metodology you can monitor pupils' progresses	2

TABLE 7. Analysis of results: PBL punti vincenti

create groups more accurately	25
better organize time	16
use a suitable classroom	14
give solutions to all exercises (e.g. through the general discussion)	11
more tutoring during group work	5
choose suitable problems	3
use different materials	3
make smaller/bigger groups	3
split the class in order to have less students	2
better involve pupils in the general discussion (e.g. make everybody talk during the general discussion)	2
have everybody work on the same problem	2
have pupils write down carefully their solutions	1
make a final test to evaluate the pupils	1
let the pupils get to the solution, never tell them the solution	1
read and explain the problems at the beginning of the session	1

TABLE 8. Analysis of results: things would like to change/to put into action

logistic (spaces, attrezzature)	51
pedagogical support (both istruzioni ma anche materiale vero e proprio)	35
economic support (materiale di consumo)	14
competenza (sia disciplinare-matematico, che pedagogica, non sempre è chiara la distinzione)	7
appoggio/collaborazione da parte dell'ambiente di lavoro	7
appropriate number of students	2
stimoli intellettuali	2
time	5

TABLE 9. Analysis of results: support needed

cooperate (do activities together)	54
share methodology/use same methodology in their lectures	19
cedere tempo e spazi se necessario	11
respect for own choiches	5
monitor pupils	1

TABLE 10. Analysis of results: support from colleagues

agree on methodology and support choice	53
economic support (fornire tempi, spazi e materiali)	27
rispetto per la libertà di scelta e l'autonomia	12
organizzare/promuovere corsi di aggiornamento	7
incentivi	1
indifferente	1

TABLE 11. Analysis of results: support from the school principal

traning (both pre-service, and in-service, no clear distinction)	49
materials (produrre materiale didattico e attività didattiche)	16
do research	15
collaborazione e coinvolgimento in attività non meglio precisata	1
finanziamenti (finanziare e promuovere le ricerche/le attività fatte dagli insegnanti in servizio)	1
focus on their attuale esperienza come studenti	11

TABLE 12. Analysis of results: support from the university

parents	48
pupils	5
musei and research centers	3
other school impiegati	2

TABLE 13. Analysis of results: who else can help?

questionnaire was nominative and students still have to face me for taking their final exams (just for the record, everybody answered "yes"). From this point of view is much more interesting to look at the answers they gave to question 5 and 6, collected in table 8, because they give more the idea on how studens "getting in the game". The figure that we feel is more indicative in this sense is the answer to question 7, although we need to point out that the analysis of the answers was more difficult than expected: actually the question asked for a "ratio" of the partition of curricular time between traditional lecturing an PBL activities, but it turned out, once again, that the language of fractions is not so such a common language as it should. Some students actually found it difficult to answer because they had no idea of how meny curricular hours they are supposed to have. We had to filter and categorize these answers. Approximately 39 studentes state that they would use PBL activities for more than 50% of the curricular time devoted to mathematics. We can add to these a further 24 that think of a weekly appointment devoted to a PBL activity. In other words a wide majority of the 94 questionnaire analized would like to use PBL activities in a significant part of the time devoted to mathematics. Some of the more explicit answers indicated that such activities should be used to introduce any new topic (in other word let the problem introduce the need for the theory). In a completely different direction a small, not marginal number of students (8) that think that PBL activities should be used at the end of each topic. Finally question 8 connect to the research on belifs, showing that these pre-service teachers acknowledge the difficulties they can meet. It is well known that "even if teachers' beliefs match curricular reform, very often the traditional nature of educational systems make it difficult for teachers to enact their espoused progressive beliefs" (Handal, 2003, p. 47); or "The context of school instruction obliges practising elementary and secondary teachers to teach traditional mathematics even when they may hold alternative views about mathematics and about mathematics teaching and learning. Parents and provessional colleagues, for example, expect teachers to teach in a traditional way. Teachers are also expected to focus on external examination, to adhere to a textbook, and to keep a low level of noise and movement in their classrooms" (Handal, 2003, p. 49). And we should not forget the episode of the "Math wars". Students' answers are collected in table 9 to 13. The cure the students put in the filling of the questionnaire answering this question, and identifying most of the point the researcher describe , denotes their consciousness. Note that "parents" was not on the given choices. In our opinion, comparing table 9 with table 6 and 8, some of the answers collected in table 9 are indeed due to the particular experience.

3.1.3. *Role of the University.* I wanted to insert a question on the role the university because currently in Italy there is a debate on what is the institution teacher evaluating should be demanded. Things such High school of education are planned, but did not start yet. In Italy (but we believe this is the case in many other European countries, e.g. see the talk by Wittmand at the Conference *The future of Mathematics Education in Europe* Wittmann (2007)) University are not involved in the editing of school textbooks, and the mathematical community seldom even get to see them (namely, matematicians look at school textbook only when they happen to have children in school age). Significantly enought the production of school teaching material is not taken in consideration for the career of university professor. The answers about the role of the university are collected in table 12, that shows the recognition of the university not only of research, but, above all, of traning too. The students' answers are seldom clear whether they mean pre-service training or in-service traning (often the two aspects are mixed). There are explicit request of the activation of a sort of helpdesk for teachers. A small, but significant, portion of the students think that the university can fill the lack of teachingn materials. Perhaps this answer is influenced by the fact that these students know the Center matematita and its proposals.

3.2. **Does and dont's.** After we asked our students to review the experience, we want to make our personal analysis. As we already stated, this was a first attempt to introduce PBL activities in University curricula. Nevertheless the constraints were very strong. The activity was planned so to give our students a small chance, but this was done at the expenses of some aspects that probably were equally important (and some expert will disagree with our implementation). The PBL session have involved a far too large class: we let free choice to the students whether to attend or not, and we ended up with more than 100 students. Whoever organized PBL activities knows too well that this was a big problem. The classroom was large enough to host all those people, but obviously the small group work caused a disturbing constant noise. The hight number of students compared to the number of tutors has meant that there was almost no tutoring at all and the groups were let to work completely on their own. The message of the importance of tutoring did not get to the students. If we look at table 6, only 3 people recognised the importance in such activities of the teacher actions. A final aspect, easy to implement, is to use a single problem for each session. The choice of giving so many problems was also due to the fact that such problems had not been tested, and we could not foresee how long students would have taken to solve them, so we wanted to provide extra material for early finishers. This had the side effect of distracting studens, as clearly emerges from table 6. The final discussion suffered from this too, as students highlighted the fact that the discussion on problems they hadn't been working was not easy to follow. At least they learned that the small group work is prerequisite to actually grasping the subject covered.
Even if the experience had clear faults, we still thinks that it was usefull for the students: at least they learned that some things should not be done. Posso però aggiungere che l'esperienza ha operato sui miei stessi belief convincendomi che lo sforzo di superare le costrizioni vale il risultato. We can add that this experience actually acted on our own beliefs: we learned that this kind of activities nicely fit in the university routine, and, even if we will have to discuss with our faculty in order to get the necessary resources, it is worth the effort. Our students deserves experiences of this kind, as "teachers' mathematical beliefs are seen as self-perpetuating within the atmosphere of a system that promotes progressive teaching but in fact helps in maintaining traditional beliefs and practices" (Handal, 2003, p. 54).

4. Final remark

We encouraged our students to learn about current theories of learning and teaching, and to actually consider *using* these in their own future classrooms, we knew that there will be obstacles which would probably prevent this from happening: "obstacles to fully implement their ideals included lack of preparation time and lack of collaboration among peers; size of room; availability of technology, materials, and money; non-supporting administration and parents; need for lengthened class periods; and personal opportunity for growth" (Handal, 2003, p. 52) University education cannot do much about these factors, but we are facing a broader cultural issue. From a certain point of view something is changing. In response to the assertion "a teacher might be motivated to provide rote-learning activities in class when that teacher knows that his or her students will ben tested on basic skills in a district proficiency exam" (Handal, 2003, p. 53) we can start answering that the international test PISA-OECD tend to assess the kind of skills that Polya put in the category of *know-how*. By analyzing the answers to the questionnaires and collecting sensations received from conversations with our students, we are lead to think that this experience has constituted a small contribution in strengthening students' beliefs in favour of a PBL teaching model. Some students asked to build a collection of the material produced by students during the course, that could be used in their future working esperience. The collection is available through a website.

Moreover, we are currently supervising a asmall portion of the student (6) that are currently proposing PBL activities to primary school children in order to produce a research for their final dissertation.

References

Barberis U (2007) Formazione matematica per l'industria. In: XVIII Congresso Unione Matematica Italiana, UMI

Bolondi G (2005) La matematica quotidiana. Quaderni a quadretti, Mimesis, Milano

Bonaiti I, Chiesa L, Lanfranchi S (2005) La formica e il miele. 60 giochi per insegnanti e ragazzi svegli. Materiali per un quaderno a quadretti, Mimesis

Caronnni P, Ciani R, Gilberti P, Rapuano M, Vitali A (2007) Conorovesciato. Un esperimento di didattica per problemi nella scuola primaria. Materiale per i Quaderni a Quadretti, Mimesis

Cazzola M (2004) 'Simmetria, giochi di specchi' in un curriculum per i futuri insegnanti. L'insegnamento della Matematica e delle Scienze integrate VOL 27/A(1):37–56

Cazzola M (2007) Fare esperienza di matematica a scuola. In: Caronnni et al (2007)

Dedò M (2001) Più matematica per chi insegna matematica. Bollettino UMI "La matematica nella società e nella cultura" Serie VIII(IV-A):247–275

Freudenthal H (1991) Revisiting Mathematics Education: China Lectures. Kluwer Academic Publishers

Freudenthal H (1994) Ripensando l'educazione matematica: lezioni tenute in Cina. La scuola, Brescia

Handal B (2003) Teachers' mathematical beliefs: A review. The Mathematics Educator 13(2):47–57

Hmelo-Silver CE (2004) Problem-based learning: What and how do students learn? Educational Psychology Review 16(3):235–266

LeBlanc JF (1982) A model for elementary teacher training in probem solving. In: Lester FKJ, Garofalo J (eds) Mathematical Problem Solving. Issues in Research

Mergendoller JR, Maxwell NL, Bellissimo Y (2006) The effectiveness of problem-based instruction: A comparative study of instructional methods adn student characteristics. The Interdisciplinary Journal of Problem-based Learning 1(2):49–69

National Research Council (1989) Everybody counts: A report to the nation on the future of mathematics education. National Academy Press, Washington, DC

Polya G (1945) How to solve it: a new aspect of mathematical method. Princeton University Press

Polya G (1962) Mathematical Discovery: On Understanding, Learning, and Teaching Problem Solving, vol Combined edition. John Wiley and Sons, 1980

Savery JR (2006) Overview of problem-based learning: Definition and distinctions. The Interdisciplinary Journal of Problem-based Learning 1(1):9–20

Schoenfeld AH (1992) Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In: Grouws D (ed) Handbook for Research on Mathematics Teaching and Learning, MacMillan, New York, pp 334–370

Schoenfeld AH (2004) The math wars. Educational policy 18(1):254–286

Taplin M, Chan C (2001) Developing problem-solving practitioners. Journal of Mathematics Teacher Education 4:285–304

Wittmann E (2007) Mathematics as a science of patterns. Talk at the conference "The future or Mathematics Education in Europe", Lisboa, 17 e 18 de Dezembro de 2007